A REVIEW OF THE LITERATURE ON WAVE PROPAGATION AND VIBRATION PROCESSES IN THREE-LAYER VISCOELASTIC PLATES WITH A FILLER

Navruzbek Qalandarov,
Muhammad Narzulloyev
Asia International University (Uzbekistan, Bukhara)

ABSTRACT

This article presents an analysis of the scientific literature devoted to the study of wave propagation and vibration processes in three-layer viscoelastic plates with a filler. The stages of development of wave propagation theory in elastic and viscoelastic media, the properties of Rayleigh, Love, Lamb and Stoneley waves, methods for obtaining dispersion relations for multilayer structures, and models accounting for the dissipative properties of materials (Voigt, Maxwell, hereditary theory) are considered. The analysis shows that the problem of wave propagation in three-layer viscoelastic plates with a filler is far from its final solution, making the development of a theoretical basis, problem statement, solution methodology, algorithm and software a relevant scientific and practical task.

Keywords: Wave propagation, three-layer plate, viscoelasticity, dispersion equation, filler layer, dissipation, Rayleigh wave, Lamb wave.

ANNOTATSIYA

Ushbu maqolada to'ldiruvchili uch qatlamli qovushqoq-elastik plastinkalarda to'lqin tarqalishi va tebranish jarayonlarini o'rganishga bag'ishlangan mavjud ilmiy adabiyotlarning tahlili keltirilgan. Elastik va qovushqoq-elastik muhitlarda to'lqin tarqalishi nazariyasining rivojlanish bosqichlari, Reley, Lyav, Lemb va Stounli to'lqinlarining xossalari, ko'p qatlamli konstruksiyalarda dispersion munosabatlarni olish usullari, hamda materiallarning dissipativ xossalarini hisobga olish modellari (Foyxt, Maksvell, irsiy nazariya) ko'rib chiqilgan. Tahlil natijalari shuni ko'rsatadiki, to'ldiruvchili uch qatlamli qovushqoq-elastik plastinkalarda to'lqin tarqalishi masalasi o'zining yakuniy yechimidan ancha uzoq bo'lib, bu yo'nalishda nazariy asos yaratish, masalani qo'yilishi, yechish uslubiyoti, algoritmi va dasturini ishlab chiqish dolzarb ilmiy-amaliy ahamiyatga ega.

Kalit so'zlar: to'lqin tarqalishi, uch qatlamli plastinka, qovushqoq-elastiklik, dispersion tenglama, to'ldiruvchi qatlam, dissipatsiya, Reley to'lqini, Lemb to'lqini.

АННОТАЦИЯ

В данной статье приведён анализ научной литературы, посвящённой исследованию волновых процессов и колебаний в трёхслойных вязкоупругих пластинах с заполнителем. Рассмотрены этапы развития теории распространения волн в упругих и вязкоупругих средах, свойства волн Рэлея, Лява, Лэмба и Стоунли, методы получения дисперсионных соотношений для многослойных конструкций, а также модели учёта диссипативных свойств материалов (Фойгта, Максвелла, наследственная теория). Результаты анализа показывают, что задача распространения волн в трёхслойных вязкоупругих пластинах с заполнителем далека от окончательного решения, вследствие чего разработка теоретической основы, постановки задачи, методики, алгоритма и программы её решения представляет собой актуальную научно-практическую задачу.

Ключевые слова: распространение волн, трёхслойная пластина, вязкоупругость, дисперсионное уравнение, слой заполнителя, диссипация, волна Рэлея, волна Лэмба.

INTRODUCTION

Worldwide, many structural elements used in engineering and construction are built as three-layer plates with a filler, and studying the dynamic state of such plates when they interact with other components through various deformable elements is one of the pressing problems of the field. Studying wave propagation in three-layer plate-like structures interacting with a supporting base makes it possible to accurately predict the resonance conditions that arise from dynamic effects in modern building structures, aircraft, shipbuilding, and machine-building equipment. In the USA, Canada, Russia and other developed countries, particular attention is being paid to the effectiveness of three-layer plates in solving vibration- and noise-suppression problems in moving vehicles.

Research into the propagation of natural waves in multilayer plates and the dynamic processes arising from external loading, together with efforts to improve such structures, is being actively pursued worldwide. Aircraft and moving vehicles largely consist of two- and three-layer structures, and assessing the dynamic state of such structures under external forces is of significant scientific and practical importance. In this context, structures are usually modeled as a three-layer plate with a filler, which requires developing methods for determining the parameters that characterize the dynamic state (wave number, frequency, phase velocity, mode shape) while accounting for the stiffness and viscoelastic properties of the filler.

The purpose of this article is to systematize the existing scientific results devoted to wave propagation and vibration processes in three-layer viscoelastic plates with a filler, to identify their strengths and shortcomings, and, on this basis, to substantiate directions for further research.

1. State of research on dynamic processes in multilayer plates

It is known from the literature that in an infinite homogeneous isotropic linearly elastic deformable medium, fundamentally different longitudinal and transverse waves exist [1, 2]. These waves also exist in multilayer plates or media, and the dynamic processes in a body can be studied through their superposition. The appearance of new geometric boundaries in a medium complicates the wave propagation process [3, 4]. In the case of plane strain, harmonic waves incident at an angle give rise to dilatational (longitudinal) and transverse (shear) waves; an incident wave generally produces two reflected waves — a process also called mode conversion [5, 6]. The incidence, reflection and refraction of a wave are determined by its angle of incidence and by the wave velocities, which depend on the material properties. In the case of deformation symmetric with respect to the mid-plane of two boundaries, two plane waves combine into a single wave propagating parallel to the boundaries; the phase velocity of this wave depends on the wavelength, and this dependence is called dispersion, found from the dispersion equation [7, 8]. The mathematical foundations of this problem are presented in [9, 10], and the propagation of waves in multilayer plates and media has since been studied extensively [11–14].

Multilayer plates are widely used in structures on elastic foundations, runways, building foundation slabs and roof structures, railways, mechanical engineering and aircraft construction. Structures used in construction are usually placed on a soil foundation, which is represented as a Winkler or Pasternak foundation model [15, 16]. When a structure is in contact with the medium through massless deformable elements (Winkler foundation), the reaction for a cylindrical shell is written as [17]:

$$q_{1r}^{k} = -\tilde{k}_{r}^{k} \cdot w^{k},\quad q_{1\theta}^{k} = 0;\quad \tilde{k}_{r}^{k}[f(t)] = k_{r0}^{k}\left[f(t) - \int_0^t R_k(t-\tau)f(\tau)d\tau\right] \tag{1}$$

where k_{r0}^k is the instantaneous bed (foundation) coefficient and $R_k(t-\tau)$ is the relaxation kernel. When the inertia of the foundation is taken into account, the Winkler model

$$q_{1r}^{k} = -\tilde{k}_{r}^{k} \cdot w^{k} - m_f \cdot \ddot{w}^{k} \tag{2}$$

takes this form (m_f is the effective mass of the foundation), while the Pasternak model is written as

$$q_{r}^{k} = -\tilde{k}_{r1}^{k} \cdot w^{k} - t_f \cdot \Delta w^{k} \tag{3}$$

where t_f is the shear coefficient and $\tilde{k}_{r1}^k$ is the operator coefficient of compression [18, 19].

Works devoted to analyzing the free vibrations of a system of isotropic plates on the ground [20, 21] exist. In recent years plates have increasingly been made of multilayer and composite materials, so studying such plates has attracted the attention of researchers worldwide. The stresses that arise in a plate and its filler are studied via the equations of elasticity theory [22, 23], and the plate equation is often derived as the Sophie Germain equation. Multilayer plates are also subject to external moving loads; such loading produces a dynamic stress-strain state in the road structure and, through repeated application, determines the development of fatigue phenomena [24, 25]. It should be noted that, although many problems have been solved, a unified theory of fatigue failure of road structures has not yet been created — this is mainly due to insufficient data on the dynamic stresses and moments involved.

The properties of localized vibrations in plates are studied, depending on the elastic properties of the material, within the framework of a semi-infinite or bounded plate problem [26, 27]; the emergence of vibrations localized near a free boundary depends on the relative dimensions of the rectangular plate [28]. Using a simplified approach based on the Cosserat model, which accounts for internal microrotations [29], the equation of motion describing traveling waves in a plate is obtained in the form:

$$\Delta u + \theta_1 \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial \vartheta}{\partial y}\right) = \frac{1}{C_t^2} \frac{\partial^2}{\partial t^2}\left[u - \gamma \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} - \frac{\partial \vartheta}{\partial x}\right)\right] \tag{4}$$

where u and ϑ are the displacements of points on the mid-surface of the plate, $St^2h = \mu/\rho$, $\theta_1 = (1+v)/(1-v)$, $\gamma = J/\rho$; μ is the shear modulus, v is Poisson's ratio, ρ is the density, and J is the moment of inertia of a material particle undergoing rotation. The problem is solved using the Lamb transformation [30, 31]

$$u = \partial\varphi/\partial x + \partial\psi/\partial y,\quad \vartheta = \partial\varphi/\partial y - \partial\psi/\partial x \tag{5}$$

and substituting (5) into (4) yields

$$C_e^2\Delta\varphi = \partial^2\varphi/\partial t^2,\quad C_e^2 = E/[(1-v^2)\rho],\quad C_t^2\Delta\psi + \gamma\, \partial^2/\partial t^2(\Delta\psi) = \partial^2\psi/\partial t^2 \tag{6}$$

Solving these equations gives the displacements, from which the strength of the plate can be assessed. Triangular plates of various shapes and boundary conditions are widely used in construction and mechanical engineering; within elasticity theory [32, 33], exact solutions have been found only for a limited number of triangular plates with identical boundary conditions, while in other cases approximate (numerical) solutions based on variational methods [34] and the finite element method [35] are used [36–40].

Multilayer plates and shells are widely used in construction and engineering; most modern structures are made of composite materials. Studying the linear and nonlinear vibrations of layered plates remains a complex problem [41, 42]; a review of the literature shows that most existing works study plates of simple geometry (rectangular, circular), while the composite structure of the plate or the dissipative properties of the layers have been studied considerably less. Approaches based on R-function theory and variational methods have been proposed for studying the geometrically nonlinear vibrations of plates of arbitrary shape [43, 44]. To find the natural vibration frequency of plates, the following formula of S.P. Timoshenko is used [45]:

$$\lambda mn = (\pi/2) \cdot (m^2/b^2 + n^2/a^2) \cdot \sqrt{D0/(\rho0h0)} \tag{7}$$

where D0 is the cylindrical stiffness of the plate, ρ0 and h0 are the density and thickness of the material, m and n are the number of half-waves along the x and y axes, and a and b are the plate dimensions.

2. Review of wave propagation theory in elastic and viscoelastic media

Studying the propagation of stress (strain) waves in elastic and viscoelastic media is one of the most important directions of wave dynamics; the fundamental differential equations of motion and their theoretical basis are given in [57–60]. In an unbounded linear elastic medium, only transverse and longitudinal (volumetric) waves propagate [60]; their frequency does not depend on the wavelength, i.e., they do not undergo dispersion. Wave propagation in a medium with a boundary was first studied by Rayleigh (1885): a wave propagating near the surface of a half-space bounded by vacuum, whose amplitude decays exponentially with depth, is called a surface (Rayleigh) wave. The Rayleigh wave does not disperse; its velocity, as Poisson's ratio varies between 0.25 and 0.4, amounts to 0.87–0.96 of the transverse wave velocity. Similarly, another non-dispersive wave type is the Stoneley wave (1924), propagating near the boundary of two elastic media; its phase velocity lies between the maximum Rayleigh velocity of the half-spaces and the minimum transverse wave velocity.

When a two-layer medium is subjected to an external dynamic force, the stress produced by that force reaches the boundary of the second layer at time $t_1 = h_1/c_1$ [61]:

$$\sigma1 = (4p/\pi D^2) \cdot \sin(h1\pi/(T0c1)) \cdot e^{(-\gamma1h1)} \tag{8}$$

The stress at the layer surface takes the form $\sigma2 = (4p/\pi D^2) \cdot \sin(h1\pi/(T0c1))$ (9), while the average stress acting on the layer is expressed as

$$\sigma avg^{(1-2)} = (2p/\pi D^2) \cdot \sin(h1\pi/(T0c1)) \cdot (1 + e^{(-\gamma1h1)}) \tag{10}$$

The displacement of the structure's surface at time $t_1 = h_1/c_1$ is determined by the equation

$$U(z=0, t1) = (2p \cdot t1 \cdot c1)/(\pi D^2E1) \cdot \sin(h1\pi/(T0c1)) \cdot (1+e^{(-\gamma1h1)}) - (\rho1c1^2t1^2)/E1 \tag{11}$$

If $t_1 > h_1/c_1$, wave reflection occurs and the stress at the contact decreases as

$$\sigma3 = (4p/\pi D^2) \cdot \sin(h1\pi/(T0c1)) \cdot e^{(-\gamma1h1)} \cdot [(1-\sqrt{E1\rho0/E0\rho1})/(1+\sqrt{E1\rho0/E0\rho1})] \tag{12}$$

It has been shown that the change in the displacement amplitude is directly related to the formation of reflected and refracted waves.

If a layer lies on a half-space, a Love wave (1911) may arise — a transverse wave propagating in the elastic half-space and the layer lying on it, which can exist only if the transverse velocity in the half-space is greater than that in the layer ($St^{(1)} < St^{(2)}$). The Love wave velocity lies between the transverse velocities of the layer and the half-space ($St^{(1)} < SLove < St^{(2)}$) and this wave is dispersive — its phase and group velocities do not coincide. As the layer thickness tends to zero, the Love wave velocity tends to the bulk wave velocity [62, 63]. In an elastic plate with free faces, the Lamb wave propagating perpendicular to the plane of the plate is studied by separating it into symmetric and antisymmetric families; at high frequencies its phase and group velocities tend to the Rayleigh wave velocity. Wave propagation in a three-layer elastic medium, or one in which one of the layers is a fluid, has been extensively studied in seismic exploration [64, 65]; in 1962 Krauklis [66] obtained, in analytical form, models in which a dispersive wave exists whose phase velocity approaches zero as the frequency tends to zero; such waves were later also identified numerically for finite elastic layers [67].

For steady-state (harmonic) wave propagation with respect to infinity, the Sommerfeld condition ensuring the uniqueness of the solution of the differential equation [68–70]

$$\lim_{r\to\infty} \sqrt{r}\cdot(\partial\varphi/\partial r + i\alpha\varphi) = 0, \quad \lim_{r\to\infty} \sqrt{r}\cdot(\partial\psi/\partial r + i\beta\psi) = 0 \qquad (13)$$

is imposed, where α and β are the longitudinal and transverse wavenumbers. D. Miklowitz (1960) reviewed the works on the propagation of elastic waves in an unbounded medium, a rod, a plate, a shell, a half-space, and a layer [71].

Wave propagation in a viscoelastic medium is studied on the basis of the Voigt, Maxwell and Kelvin models [72–74]. One of the main properties of viscoelastic materials (polymers, composites, concretes, rocks, high-temperature metals) is creep (the increase of strain under constant stress) and relaxation (the decrease of stress under constant strain). Structural metals behave as elastic bodies at ordinary temperatures (from −20 °C to +20 °C), and exhibit viscoelastic properties above +200 °C [75, 76]; plastics behave close to an elastic body at 0 °C, but clearly exhibit viscoelastic properties at +50 °C [77–79]. Because these classical models do not account for hereditary processes in materials, the Boltzmann–Volterra integral relation is used instead [80, 81]. One of the central problems of dynamic elasticity theory is the study of wave propagation in waveguides of various cross-sections [82–86]; for waveguides with an annular cross-section, this problem reduces to solving a dispersion equation [87–89], and corresponding results also exist for waveguides with non-circular (non-classical) cross-sections [90, 91].

Studying the dissipative properties of wave propagation in various mechanical systems and waveguides is of particular scientific interest. Energy dissipation refers to the conversion of mechanical energy into another form of energy (for example, heat); if the medium is connected to infinity, dissipation also occurs due to the wave-like outflow of energy to infinity. There are works devoted to studying wave propagation in various viscoelastic mechanical systems accounting for dissipative properties [92, 93], and works deriving equations for determining dynamic stresses in plates and shells on the basis of differential and variational approaches [94, 95]. The free and forced vibrations of dissipative mechanical systems are presented in [98, 99], where various models (Voigt, Maxwell, hereditary theory) are used to describe viscosity;

there are also numerous works on the use of damping coatings to reduce the periodic vibration or vibration of structures in engineering [100, 101]. The equations of motion of practical problems have been solved by numerical and analytical methods [102, 103]. Furthermore, the problem of the non-orthogonality of the Lamb wave through the thickness of a layer (i.e., the scalar product of two arbitrary distinct mode shapes is not zero) is also relevant for a viscoelastic plate, and a number of works are devoted to this issue [104, 105].

CONCLUSION

The review shows that, despite the large number of scientific works on wave propagation in viscoelastic plate-like and cylindrical waveguides, the problem of wave propagation in three-layer viscoelastic plates with a filler is still far from its final solution. In particular:

— most existing works cover single-layer or two-layer plates of simple geometry (rectangular, circular), whereas three-layer, filled, viscoelastic structures have been studied considerably less;

— the methodology for obtaining dispersion relations accounting for various contact conditions between layers (rigid bonding, slip, separation) has not been fully developed;

— methods for reducing the resonance amplitude of a mechanical system, accounting for the dissipative (viscous) properties of materials on the basis of hereditary theory, have not been sufficiently studied.

Therefore, formulating the problems of wave propagation and vibration in three-layer viscoelastic plates with a filler, and developing the methodology, algorithm and software for solving them, remains an urgent scientific and practical task. Further research in this direction is presented in detail in the subsequent articles prepared by the authors — covering the problem statement and solution methodology, exact results for wave propagation in three-layer rods and plates, and the dynamic stress-strain state under harmonic loading.

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