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FORCED VIBRATIONS OF ROTARY SHAFTS MOUNTED ON VISCOELASTIC SUPPORTS AND THE RESULTS OF THEIR INDUSTRIAL IMPLEMENTATION

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ABSTRACT

This paper examines the problem of forced linear vibrations of a system of coaxial rotors mounted on viscoelastic supports and presents the results of the practical implementation of the research findings in the grain-milling industry. A non-homogeneous system of integro-differential equations, constrained by servo-connections, is derived on the basis of Lagrange's equations of the second kind and solved by the Gauss method. A finite-element analysis carried out in the SOLIDWORKS software package shows 95-97% agreement between the theoretical results and actual production data. It is demonstrated that the localization of centrifugal rotary-shaft production leads to a 2-6% reduction in metal consumption, an 11-13% reduction in energy consumption, a 12.5-14.5% increase in the service life of the shafts and their drives, and an expected annual economic effect of 82.6 million soums.

Keywords: Rotary shaft, forced vibrations, coaxial rotor, finite-element method, SOLIDWORKS, metal consumption, economic efficiency.

1. INTRODUCTION

Rotary shafts used in the grinding of grain products constitute an important structural component of the machine-building and food industries. During operation, such shafts are subjected to harmonic and non-stationary dynamic loads, which necessitates a theoretically justified assessment of their stress-strain state as well as optimization of their structural parameters. Building on the mathematical model and solution methodology previously developed by the author, this paper solves the problem of forced vibrations and analyzes the results of the practical implementation of the theoretical findings at the Bukhara Mechanical Repair Plant JSC and at Tashkent State Technical University named after I. Karimov.

The purpose of the research is to study the forced vibrations of a deformable shaft structure mounted on viscoelastic supports, to determine the amplitudes characterizing the stress-strain state as a function of the support parameters, and to implement the obtained results in industrial production.

The research tasks include studying the forced vibrations of the deformable shaft structure mounted on viscoelastic supports, determining the amplitudes that characterize the stress-strain state as a function of the support parameters, numerically verifying the theoretical results in SOLIDWORKS, and subjecting the developed recommendations to practical testing under production conditions. The study employs methods of deformable solid mechanics, mathematical modeling, the finite-element method, and computer-aided engineering software (SOLIDWORKS).

The scientific novelty of this stage of the study lies in the formulation and solution of the forced-vibration problem for a system of coaxial viscoelastic rotors constrained by servo-connections, and in establishing the dependence of the resonance amplitude and its position on the frequency axis on the placement of the disk along the shaft. The practical novelty consists in the numerical verification of the theoretical model by finite-element analysis and in the industrial approbation of the developed recommendations under real production conditions, including quantitative confirmation of their technical and economic effect.

2. MATERIALS AND METHODS: MATHEMATICAL MODEL OF FORCED VIBRATIONS

A system of coaxial viscoelastic rotors (rotor 1 and rotor 2) is defined by corresponding coordinates. In the undeformed state, the axes of the rotors coincide, while the supports (points C, D and B) are assumed to be absolutely rigid. Using Lagrange's equations of the second kind, the system is described by the following matrix form of a linear system of integro-differential equations:

$M\ddot{q} + L\dot{q} + C_0 y = 0$ (1)

where M, L and C_0 are the mass, damping and stiffness matrices, and y is the vector of generalized displacements. If the motion of the shaft is constrained by servo-connections, the system of equations becomes non-homogeneous:

$M\ddot{q} + L\dot{q} + C_0 y = R(y)$ (2)

where R(y) is the reaction force of the servo-connection. The forced-vibration problem is solved for a prescribed law of rotation of the first rotor. To determine how the frequency of the shaft system depends on the support stiffness, 20 points are selected along the shaft. The parameters of the three-parameter weakly singular Koltunov-Rzhanitsyn kernel, which characterizes the viscosity of the medium, are adopted, and the viscosity parameters of the first and second supports are assumed to be equal.

The dependence of the vibration amplitude on the external-force frequency was analyzed for a disk mounted on the shaft at various positions (l/4, l/3 and 2l/3 of the shaft length). The results show that the position of the disk substantially affects both the magnitude of the resonance amplitude and its location on the frequency axis (amplitude values ranging from A = 0.041 to A = 0.15), indicating that the optimal placement of disks along the shaft must be carefully considered in engineering practice.

3. RESULTS: FINITE-ELEMENT ANALYSIS IN SOLIDWORKS

To verify the theoretical results, the rotary-shaft structure was modeled in the SOLIDWORKS software package using the finite-element method. The following geometric dimensions were adopted for a shaft made of 40St steel (Table 1):

Table 1. Geometric dimensions of the shaft

Element	Nodes	Inner diameter, m	Outer diameter, m	Length, m
1	1-2	0.055	0.070	0.049
2	2-3	0.055	0.070	0.049
3	3-4	0.055	0.070	0.045
4	4-5	0.055	0.070	0.0515
5	5-6	0.055	0.070	0.0515
6	6-7	0.055	0.070	0.0480
7	7-8	0.055	0.070	0.0490

The deformation of the shaft under a distributed force and torque was studied. The total linear strain components (EPSX, EPSY, EPSZ, EPSXY, EPSXZ, EPSYZ) were found to range from 0.01528 to 2587.38 (in microstrain units), while the strain intensity (EPSINT) ranged from 0 to 0.011212. The first three natural vibration mode shapes were plotted, and their nodal points were identified.

The numerical results also made it possible to identify the critical cross-sections of the shaft where the maximum deformation and stress values occur. These sections were typically located near the supports and in the areas where the disks are mounted, indicating that additional reinforcement or optimization of the structural dimensions is required in these zones during practical design. Further analysis in SOLIDWORKS showed that modifying the structural dimensions of the rotary shafts significantly reduced their deflection and loading, confirming that the theoretical model can also be effectively applied at the practical design stage.

Increasing the linear speed of the fast-rotating shaft from 4 to 10 m/s was found to improve the degree of grinding at all stages of the process, while a speed of 5-6 m/s is recommended for other processes. At the same time, increasing the linear speed above 6 m/s causes the grain to spin without being ground and increases energy consumption; therefore, a linear speed range of 4-5 m/s is recommended for grinding processes. The service life of existing rotary shafts was verified in SOLIDWORKS and found to match actual production performance with 95-97% accuracy, confirming the practical reliability of the developed calculation methodology.

These findings underline the importance of combining analytical modeling with computer-aided finite-element verification when designing shaft-support systems for industrial applications: while the analytical model provides a physically transparent description of the governing dynamic phenomena, the finite-element simulation offers a detailed, geometry-specific validation that captures local stress concentrations which are difficult to obtain from the reduced-order analytical model alone. The close agreement between the two approaches observed in this study therefore strengthens confidence in both the theoretical formulation and its numerical implementation described here.

4. INDUSTRIAL IMPLEMENTATION RESULTS

On the basis of the obtained theoretical and numerical results, shafts mounted on viscoelastic supports underwent pilot-industrial testing at the Bukhara Mechanical Repair Plant JSC. As a result, methods for reducing the loads on the shafts were developed; the theoretical foundations of the casting composition and processing required for localizing centrifugal rotary-shaft production were established; and recommendations for a new type of shaft design, based on the dynamic parameters necessary for rotary-shaft production, were formulated. In addition, at Tashkent State Technical University named after I. Karimov, the hard layer of the shafts used in wheat grinding was determined to be alloyed steel grade 8620 (AISI 8620), together with the corresponding sapphire material (certificate of the Food Industry Association of Uzbekistan No. 04-60/06-25 dated 04.06.2025).

The achieved practical results are summarized in Table 2:

Table 2. Summary of practical implementation results

Indicator	**Practical outcome**
Localization of rotary-shaft production	Metal consumption reduced by 2-6%
Reduction of shaft mass	Service life of rotary shafts and drives increased by 12.5-14.5%
Implementation of the developed technology	Energy consumption reduced by 11-13%
Expected economic effect	82.6 million soums per year
Verification in SOLIDWORKS	95-97% agreement with real production data

As part of developing the theoretical foundations for reducing the loads on rotary shafts, an explicit mathematical solution was obtained for the dependence of bending (deformation) on the inner diameter of the shaft, providing designers with a means of purposefully reducing loads through the optimal selection of shaft diameter.

5. DISCUSSION

The economic and technical significance of the implemented results is manifested in several respects. First, the 2-6% reduction in metal consumption directly lowers raw-material costs and enables more rational use of local casting production capacity. Second, the reduction in shaft mass decreases the inertial forces acting on the drives and bearings, which in turn extends their service life by 12.5-14.5%. Third, the 11-13% reduction in energy consumption improves the overall energy efficiency of the enterprise, in line with the priority directions of the Republic of Uzbekistan for energy and resource conservation.

The reduction in the required bearing series and the simplified maintenance of the deformable shaft reduce repair and maintenance costs. Moreover, the identified optimal linear-speed range (4-5 m/s) and the recommended structural parameters of the rotary shafts can also be applied at other grain-milling enterprises, extending the significance of the results beyond a single plant. The expected economic effect of 82.6 million soums per year was calculated for a single enterprise; wider implementation of the technology at other food-industry enterprises could substantially increase the overall economic benefit.

The reliability of the obtained results is confirmed by the 95-97% agreement between the theoretical calculations and the finite-element analysis performed in SOLIDWORKS, as well as by the official certification of the practical test results by the Food Industry Association of Uzbekistan. This indicates that the developed methodology and structural recommendations can be confidently applied at other machine-building and food-industry enterprises.

Directions for further research in this area include the use of alternative alloys and composite materials for rotary-shaft manufacturing, as well as the integration of digital-twin technologies for real-time condition monitoring. Such developments would create new scientific and practical opportunities for further extending shaft service life and reducing energy consumption.

6. CONCLUSION

As a result of the conducted research, the problem of forced linear vibrations of a system of coaxial rotors mounted on viscoelastic supports was solved using Lagrange's equations and the Gauss method, and the influence of disk position on the amplitude-frequency characteristics was determined. The finite-element analysis performed in SOLIDWORKS showed that the theoretical results agree with actual production data with 95-97% accuracy. Implementation of the developed methodology and recommendations at the Bukhara Mechanical Repair Plant JSC and at Tashkent State Technical University named after I. Karimov resulted in a 2-6% reduction in metal consumption, an 11-13% reduction in energy consumption, a 12.5-14.5% increase in the service life of the shafts and their drives, and an expected annual economic effect of 82.6 million soums. The obtained results provide a scientific and practical basis for the further development of resource-saving methods for the design and production of rotary shafts. At the same time, the developed approach is also of practical value for the design and modernization of shaft structures used in mechanical engineering, aviation-space and other industries, thereby contributing to the priority national directions for the development of energy- and resource-saving equipment and technologies.

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