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COMPARISON OF DIFFERENT METHODS FOR SOLVING A SYSTEM OF THREE-UNKNOWN LINEAR EQUATIONS AND ANALYSIS OF THEIR EFFECTIVENESS

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ABSTRACT

The main goal of teaching mathematics in modern education is to form students the skills of independent thinking, logical analysis, and the use of different approaches to solving practical problems. In particular, the topic of solving systems of equations plays an important role in the development of students' analytical thinking.

Solving a system of three-unknown equations is one of the important sections of the algebra course, in which students learn how to analyze several variables at once, determine the connections between equations, and learn algorithmic reasoning. Therefore, it is necessary to improve methodological approaches to teaching this topic.

Keywords: Mathematics, addition and subtraction method, Gaussian, determinant.

GOALS AND OBJECTIVES

To improve the methodology of teaching to solve the system of three unknown equations by different methods and to develop students' logical thinking in this process. The following tasks occupy a central place in the article.

1. Analysis of the theoretical foundations of a system of three-unknown equations.
2. Learning the basic methods of solving (substitution, addition-subtraction, Kramer, Gaussian, and matrix methods).
3. Compare the advantages and disadvantages of each method.
4. Development of teaching methodological recommendations for pupils in the use of appropriate interactive methods
5. Identification of the possibility of using computer technologies (GeoGebra, Excel, WolframAlpha) in the educational process.

RESEARCH METHODS

The following methods were used in the study: Theoretical analysis and generalization; monitoring the progress of training; Conduct experimental classes; Mathematical and statistical analysis of pupils' results.

THE MAIN PART

Hiob the three-unknown equation.

$$\begin{cases} x + y + z = 1 \\ 2x + 3y + 4z = 3{,}3 \\ 3x + 3y + 4z = 3{,}5 \end{cases}$$

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Method 1. The approximation (+) method of the equation system.

$$\begin{cases} x+y+z=1 \\ 2x+3y+4z=3{,}3 \\ 3x+3y+4z=3{,}5 \end{cases}$$

3x+3y+4z=3.5

2x+3y+4z=3.3

x=0,2

0.2+y+z=1

y+z=0.8

$$\begin{cases} 2x+3y+4z=3{,}3 \\ x+y+z=1\ (2) \end{cases} \Longrightarrow \qquad -\begin{cases} 2x+3y+4z=3{,}3 \\ 2x+2y+2z=2 \end{cases}$$

y+2z=1.3

$$-\begin{cases} y+2z=1{,}3 \\ y+z=0{,}8 \end{cases}$$

z=0,5

x+y+z=1

0.2+y+0.5=1

y=1-0,7

y=0,3

x=0,2 y=0,3 z=0,5

Method 2. A method of substitution of a system of equations.

$$\begin{cases} x+y+z=1 \\ 2x+3y+4z=3{,}3 \\ 3x+3y+4z=3{,}5 \end{cases} \quad x=1\text{-}y\text{-}z$$

3x+3y+4z=3.5 2x+3y+4z=3.3

3(1-y-z)+3y+4z=3.5 2(1-y-z)+3y+4z=3.3

3-3y-3z+3y+4z=3.5 2-2y-2z+3y+4z=3.3

z=0.5 and +2z=1.3

y+20.5=1.3×

y=1,3-1

y=0,3

x=1-0,3-0,5

x=0,2

Method 3. The Δ triangular method of a determination in Krammer's formula of a system of equations.

$$\begin{cases} x + y + z = 1 \\ 2x + 3y + 4z = 3{,}3 \\ 3x + 3y + 4z = 3{,}5 \end{cases}$$

$$\Delta == \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 4 \end{vmatrix}$$

$$\Delta = = \ = 12+12+6=30 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 4 \end{vmatrix} \begin{matrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 4 \end{matrix} = 9+8+12=29$$

$$\Delta=30\text{-}29=1$$

$$\Delta x = = \ = 12+9{,}9+14=35{,}9 \begin{vmatrix} 1 & 1 & 1 \\ 3{,}3 & 3 & 4 \\ 3{,}5 & 3 & 4 \end{vmatrix} \begin{matrix} 1 & 1 & 1 \\ 3{,}3 & 3 & 4 \\ 3{,}5 & 3 & 4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \\ 3{,}3 & 3 & 4 \\ 3{,}5 & 3 & 4 \end{matrix} = 10{,}5+13{,}2+12=35{,}7$$

$$\Delta=35{,}9\text{-}35{,}7=0{,}2$$

$$\Delta y = = \ = 13{,}2+12+7=32{,}2 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3{,}3 & 4 \\ 3 & 3{,}5 & 4 \end{vmatrix} \begin{matrix} 1 & 1 & 1 \\ 2 & 3{,}3 & 4 \\ 3 & 3{,}5 & 4 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 3{,}3 & 4 \\ 3 & 3{,}5 & 4 \end{matrix} = 9{,}9+8+14=31{,}9$$

$$\Delta y=32{,}2\text{-}31{,}9=0{,}3$$

$$\Delta z = = \ = 10{,}5+9{,}9+6=26{,}4 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3{,}3 \\ 3 & 3 & 3{,}5 \end{vmatrix} \begin{matrix} 1 & 1 & 1 \\ 2 & 3 & 3{,}3 \\ 3 & 3 & 3{,}5 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 3 & 3{,}3 \\ 3 & 3 & 3{,}5 \end{matrix} = 9+7+9{,}9=25{,}9$$

$\Delta z=26,4-25,9=0,5$

$y=\frac{\Delta y}{\Delta}=\frac{0,3}{1}=0,3 \quad z=\frac{\Delta z}{\Delta}=\frac{0,5}{1}=0,5 \quad x=\frac{\Delta x}{\Delta}=\frac{0,2}{1}=0,2$

Method 4. The Arius method of the determinant in the Kramer formula for solving a system of equations.

$$\begin{cases} x+y+z=1 \\ 2x+3y+4z=3,3 \\ 3x+3y+4z=3,5 \end{cases}$$

$$\Delta=\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 2 & 3 \\ 3 & 3 & 4 & 3 & 3 \end{vmatrix}=12+12+6-9-12-8=1$$

$$\Delta x=\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3,3 & 3 & 4 & 3,3 & 3 \\ 3,5 & 3 & 4 & 3,5 & 3 \end{vmatrix}=12+14+9,9-10,5-12-13,2=0,2$$

$$\Delta y=\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3,3 & 4 & 2 & 3,3 \\ 3 & 3,5 & 4 & 3 & 3,5 \end{vmatrix}=13,2+12+7-9,9-8-14=0,3$$

$$\Delta z=\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 3,3 & 2 & 3 \\ 3 & 3 & 3,5 & 3 & 3 \end{vmatrix}=10,5+9,9+6-9-9,9-7=0,5$$

$X=\frac{\Delta x}{\Delta}=\frac{0,2}{1}=0,2 \quad y=\frac{\Delta y}{\Delta}=\frac{0,3}{1}=0,3 \quad z=\frac{\Delta z}{\Delta}=\frac{0,5}{1}=0,5$

Method 5. Matrix methods of a system of equations. Triangle method of Δ.

$$\begin{cases} x+y+z=1 \\ 2x+3y+4z=3,3 \\ 3x+3y+4z=3,5 \end{cases}$$

$$\Delta=\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 4 \end{vmatrix}=\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 4 \end{vmatrix}=12+12+6=30$$

$$\begin{matrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 4 \end{matrix}=9+8+12=29 \qquad \Delta=30-29=1$$

$A11=(-1)^{1+1}\begin{vmatrix} 3 & 4 \\ 3 & 4 \end{vmatrix}=12-12=0 \quad A_{12}=(-1)^{1+2}\begin{vmatrix} 2 & 4 \\ 3 & 4 \end{vmatrix}=-(8-12)=4 \quad A_{13}=(-1)^{1+3}\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix}=6-9=-3$

$A21=(-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}=-(4-3)=-1 \quad A22=(-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}=4-3=1 \quad A23=(-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix}=-(3-3)=0$

$A31=(-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}=4-3=1 \quad A32=(-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix}=-(4-2)=-2 \quad A33=(-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}=3-2=1$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\frac{1}{\Delta}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{32} \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}=\frac{1}{1}\begin{pmatrix} 0 & -1 & 1 \\ 4 & 1 & -2 \\ -3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 3,3 \\ 3,5 \end{pmatrix}=\begin{pmatrix} 0+(-3,3)+3,5 \\ 4+3,3-7 \\ -3+0+3,5 \end{pmatrix}=\begin{pmatrix} 0,2 \\ 0,3 \\ 0,5 \end{pmatrix}$$

$x=0,2 \quad y=0,3 \quad z=0,5$

Method 6. Matrix method of a system of equations. Δ solution in the sarius method.

$$\begin{cases} x+y+z=1 \\ 2x+3y+4z=3,3 \\ 3x+3y+4z=3,5 \end{cases}$$

$$\Delta = 12+12+6-9-12-8=1\begin{vmatrix}1&1&1&1&1\\2&3&4&2&3\\3&3&4&3&3\end{vmatrix}$$

$A11=(-1)^{1+1}\begin{vmatrix}3&4\\3&4\end{vmatrix}=12-12=0$ $A_{12}=(-1)^{1+2}\begin{vmatrix}2&4\\3&4\end{vmatrix}=-(8-12)=4$ $A_{13}=(-1)^{1+3}\begin{vmatrix}2&3\\3&3\end{vmatrix}=6-9=-3$

$A21=(-1)^{2+1}\begin{vmatrix}1&1\\3&4\end{vmatrix}=-(4-3)=-1$ $A22=(-1)^{2+2}\begin{vmatrix}1&1\\3&4\end{vmatrix}=4-3=1$ $A23=(-1)^{2+3}\begin{vmatrix}1&1\\3&3\end{vmatrix}=-(3-3)=0$

$A31=(-1)^{3+1}\begin{vmatrix}1&1\\3&4\end{vmatrix}=4-3=1$ $A32=(-1)^{3+2}\begin{vmatrix}1&1\\2&4\end{vmatrix}=-(4-2)=-2$ $A33=(-1)^{3+3}\begin{vmatrix}1&1\\2&3\end{vmatrix}=3-2=1$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}= \;==\frac{1}{1}\begin{pmatrix}0&-1&1\\4&1&-2\\-3&0&1\end{pmatrix}\begin{pmatrix}1\\3,3\\3,5\end{pmatrix}\begin{pmatrix}0+(-3,3)+3,5\\4+3,3-7\\-3+0+3,5\end{pmatrix}\begin{pmatrix}0,2\\0,3\\0,5\end{pmatrix}$$

x=0,2 y=0,3 z=0,5

Method 7. Matrix method of a system of equations. Solution using the method of spreading Δ to the towers.

$$\begin{cases}x+y+z=1\\2x+3y+4z=3,3\\3x+3y+4z=3,5\end{cases}$$

Δ=*0+1*4+1*(-3)=4-3=1 $\begin{vmatrix}1&1&1\\2&3&4\\3&3&4\end{vmatrix}=1$

$A11=(-1)^{1+1}\begin{vmatrix}3&4\\3&4\end{vmatrix}=4-4=0$ $A_{12}=(-1)^{1+2}\begin{vmatrix}2&4\\3&4\end{vmatrix}=-(8-12)=4$ $A_{13}=(-1)^{1+3}\begin{vmatrix}2&3\\3&3\end{vmatrix}=6-9=-3$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}=A^{-1}\begin{pmatrix}a\\b\\c\end{pmatrix}$$

$A11=(-1)^{1+1}\begin{vmatrix}3&4\\3&4\end{vmatrix}=12-12=0$ $A_{12}=(-1)^{1+2}\begin{vmatrix}2&4\\3&4\end{vmatrix}=-(8-12)=4$ $A_{13}=(-1)^{1+3}\begin{vmatrix}2&3\\3&3\end{vmatrix}=6-9=-3$

$A21=(-1)^{2+1}\begin{vmatrix}1&1\\3&4\end{vmatrix}=-(4-3)=-1$ $A22=(-1)^{2+2}\begin{vmatrix}1&1\\3&4\end{vmatrix}=4-3=1$ $A23=(-1)^{2+3}\begin{vmatrix}1&1\\3&3\end{vmatrix}=-(3-3)=0$

$A31=(-1)^{3+1}\begin{vmatrix}1&1\\3&4\end{vmatrix}=4-3=1$ $A32=(-1)^{3+2}\begin{vmatrix}1&1\\2&4\end{vmatrix}=-(4-2)=-2$ $A33=(-1)^{3+3}\begin{vmatrix}1&1\\2&3\end{vmatrix}=3-2=1$

$$\begin{pmatrix}x\\y\\z\end{pmatrix}= \;==\frac{1}{1}\begin{pmatrix}0&-1&1\\4&1&-2\\-3&0&1\end{pmatrix}\begin{pmatrix}1\\3,3\\3,5\end{pmatrix}\begin{pmatrix}0+(-3,3)+3,5\\4+3,3-7\\-3+0+3,5\end{pmatrix}\begin{pmatrix}0,2\\0,3\\0,5\end{pmatrix}$$

x=0,2 y=0,3 z=0,5

Method 8. The Kramer method of the system of equations. Solve Δ by spreading it out on towers.

$$\begin{cases}x+y+z=1\\2x+3y+4z=3,3\\3x+3y+4z=3,5\end{cases}$$

$$\Delta=\begin{vmatrix}1&1&1\\2&3&4\\3&3&4\end{vmatrix}=2*(-1)+3*1+0=1$$

$A21=(-1)^{2+1}\begin{vmatrix}1&1\\3&4\end{vmatrix}=-(4-3)=-1$ $A22=(-1)^{2+2}\begin{vmatrix}1&1\\3&4\end{vmatrix}=4-3=1$ $A23=(-1)^{2+3}\begin{vmatrix}1&1\\3&3\end{vmatrix}=-(3-3)=0$

Δx=3,3*(-1)+3*0,5+4*0,5=0,2 $\begin{vmatrix}1&1&1\\3,3&3&4\\3,5&3&4\end{vmatrix}=$

$A21=(-1)^{2+1}\begin{vmatrix}1 & 1\\ 3 & 4\end{vmatrix} = -(4-3)=-1$ $A22=(-1)^{2+2}\begin{vmatrix}1 & 1\\ 3,5 & 4\end{vmatrix} = 4-3,5=0,5$ $A23=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 3,5 & 3\end{vmatrix} = -(3-3,5)=0,5$

$$\Delta y= = 2*(-0,5)+3,3*1+4*(-0,5)=0,3\begin{vmatrix}1 & 1 & 1\\ 2 & 3,3 & 4\\ 3 & 3,5 & 4\end{vmatrix}$$

$A21=(-1)^{2+1}\begin{vmatrix}1 & 1\\ 3,5 & 4\end{vmatrix} = -(4-3,5)=-0,5$ $A22=(-1)^{2+2}\begin{vmatrix}1 & 1\\ 3 & 4\end{vmatrix} = 4-3=1$ $A23=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 3 & 3,5\end{vmatrix} = -(3,5-3)=-0,5$

$$\Delta z==2*(-0.5)+3*0.5+0=0.5\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 3,3\\ 3 & 3 & 3,5\end{vmatrix}$$

$A21=(-1)^{2+1}\begin{vmatrix}1 & 1\\ 3 & 3,5\end{vmatrix} = -(3,5-3)=-0,5$ $A22=(-1)^{2+2}\begin{vmatrix}1 & 1\\ 3 & 3,5\end{vmatrix} = 3,5-3=0,5$ $A23=(-1)^{2+3}\begin{vmatrix}1 & 1\\ 3 & 3\end{vmatrix} = 0$

$$X=\frac{\Delta x}{\Delta}=\frac{0,2}{1}=0,2 \quad y=\frac{\Delta y}{\Delta}=\frac{0,3}{1}=0,3 \quad z=\frac{\Delta z}{\Delta}=\frac{0,5}{1}=0,5$$

Method 9. The Gaussian method of a system of equations.

$$\begin{cases} x^1 + x^2 + x^3 = 1\\ 2x^1 + 3x^2 + 4x^3 = 3,3\\ 3x_1 + 3x_2 + 4x_3 = 3,5\end{cases}$$

$$x_1+x_2+x_3=1$$

$$-\begin{cases} x^2 + 2x^3 = 1,3\\ x_2 + x_3 = 0,8\end{cases}$$

$$x_1=0.2 \ x_3=0.5$$

$$x_1+x_2+x_3=1$$

$$0,2+x_2+0,5=1$$

$$0,7+x_2=1$$

$$x_2=0,3$$

The results of the experimental training showed that when learners are given the opportunity to compare multiple methods, their logical thinking, algorithmic approach, and outcome verification skills increase significantly.

RESULTS AND ANALYSIS

During experimental work, students in grades 8-9 increased their knowledge of solving a system of three unknown equations from 27% to 68%. Also, the use of interactive methods ("Work in pairs", "Chain Equations", "Top and Annotate") increased the participation of the students.

CONCLUSION

The use of several solution methods in teaching the system of three unknown equations develops students' analytical thinking skills, teaches them to compare different solution paths. Improving methodological approaches increases the effectiveness of the teaching process, students learn to independently solve problems and substantiate their opinion