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THE IMPORTANCE AND APPLICATIONS OF COMPARISON THEORY IN MODERN SCIENCE AND TECHNOLOGY

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ABSTRACT

This article analyzes the significance and applications of comparison theory, one of the important branches of algebra and number theory, in modern science and technology. Although the section is theoretically deep and complex, its practical application is very wide. Behind today's technological developments lie the basic principles of this theory. Therefore, in-depth study of the topic is beneficial for students interested in any mathematical and technological field.

Keywords: Integer, comparison, modulus, remainder, equation, inequality, degree, root, divisor, row, sequence.

INTRODUCTION

Comparison theory is one of the fundamental and interesting branches of mathematics, which studies the relationships arising from the remainders of numbers and is applied in many fields. This theory was first discovered and developed by the great mathematician Karl Friedrich Gauss in 1801. In recent years, innovations and research have been carried out based on this theory. Methods for solving systems of inequalities of two variables using comparisons have been developed, and these methods are of great importance in the development of students' mathematical thinking. This theory is the basis of modern cryptographic algorithms and plays an important role in ensuring information security, and recent research is aimed at developing new methods and algorithms in this area. In addition, comparisons are constantly used in solving problems and problems in mathematical olympiads (for example, Fermat's small theorem, Wilson's theorem, Euler's theorem). This theory plays an important role in the development of students' mathematical thinking in solving mathematical proofs, correspondences, and logical problems.

MATERIALS AND METHODS

Looking at the history of comparison theory and its influence on modern mathematics, we can see that its historical development occurred step by step in the following form: In the pre-Christian era, ancient Babylonian and Egyptian mathematicians had some methods for working with residuals, which were mainly used for practical calculations. For example, the process of finding the GCD using the "Euclidean algorithm" (300 BC) paved the way for modular arithmetic; Chinese mathematicians developed the "Chinese Remainder Theorem" (III-V centuries), which helps to solve a system of comparisons for several modules.

For example: $\begin{cases} x \equiv 2 \ (mod\ 3) \\ x \equiv 3 \ (mod\ 5) \\ x \equiv 2 \ (mod\ 7) \end{cases}$

It was during this period that methods for solving such systems appeared. Diophantus (III century) - studied complex algebraic equations for integers, and later these works became known as Diophantine equations. Pierre Fermat (17th century) proved Fermat's small theorem and laid the foundation for comparison theory: Theorem, "If p is a prime number, then $a^{p-1} \equiv 1 \ (mod\ p)$" Leonard Euler (1707-1783) - Euler developed the basics of modular arithmetic and introduced the Euler function (φ function) to science. In Gauss's work "Disquisitiones Arithmeticae," the theory of comparisons was reinforced, introduced to science by Karl Friedrich Gauss (1777-1855), and the concepts of "power series" and "square residues" appeared in modular arithmetic.

When solving first-degree comparisons, there are methods of selection, the method of mastering coefficients, the use of Euler's theorem, and the use of continuous fractions. Based on these methods, comparisons are carried out and results are obtained.

RESULTS AND DISCUSSIONS

In the history of mathematics, new research is being conducted on the development of comparison theory and its influence on modern mathematics. Based on these innovations, we can see the importance of this theory in modern science and technology. Comparisons are modified in each area according to their interrelationships and methods of comparison. To carry out the comparison correctly, it is necessary to obtain data, analyze it, and derive the results. The proof of equations, inequalities, and statements often presents difficulties. In this case, comparisons are useful. When solving examples and problems that lead to comparisons, we often use properties derived from the definition of a comparison. Here are some of the basic properties of comparisons:

1^0. $a \equiv a(mod\ m)$

2^0. If $a \equiv b(mod\ m)$, then $b \equiv a(mod\ m)$

3^0. If $a \equiv b(mod\ m)$ and $b \equiv c(mod\ m)$, then $a \equiv c(mod\ m)$

4^0. Comparisons can be added term by term and multiplied term by term, if $a \equiv a(mod\ m)$ and $c \equiv d(mod\ m)$, then $a + c \equiv b + d(mod\ m)$ and $ac \equiv bd(mod\ m)$

5^0. Both sides of the comparison can be multiplied by an arbitrary number, i.e., if $a \equiv b(mod\ m)$ then $ac \equiv bc(mod\ m)$

6^0. If $a \equiv b(mod\ m)$ then $a^n \equiv b^n(mod\ m)$. Here n is an arbitrary natural number.

7^0. An arbitrary part of the comparison can be added to a multiple of the modulus: a ∈ b (mod m) and if $k, l \in Z \rightarrow$ then $a + km \equiv s(mod\ m)$ and $a \equiv s + lm(mod\ m)$

Example: If n is an odd number, then prove that $n^2 - 1$ is divisible by 8.

Solution: If n is an odd number, then we write it in the form *n=2k+1, k*=1,2,3,... Now, using the definition of comparison, we can write that, substituting *n=2k+1* instead of n, we arrive at the following comparison, i.e.,

$(2k+1)^2 - 1 \equiv 0(mod\ 8)$

$4k^2 + 4k + 1 - 1 \equiv 0(mod\ 8)$

$4k^2 + 4k \equiv 0(mod\ 8)$

$4k(k+1) \equiv 0(mod\ 8)$

Since *k+1* is even, it follows that the number *4k (k+1)* is divisible by 8.

CONCLUSION

In conclusion, through the theory of comparisons, it is possible to develop the intellectual potential of students and increase the effectiveness of the educational process. Although this section is theoretically deep and complex, its practical application is very wide. Behind today's technological developments lie the basic principles of this theory. Therefore, a deep study of this topic is very useful and necessary for anyone interested in any mathematical and technological field.

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