PROPAGATION OF OSCILLATIONS AND CHANGE IN PHASE VELOCITY OF A THREE-LAYER ROD

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Consider a three-layer structure consisting of a bar or a beam (Figure .1). Let the structure be infinite on both sides. Let the member material be elastic. Suppose that the bending of the structure occurs in the plane xy. If the problem of plane deformation of the plate is considered, then the Poisson ratio ν is equal to $\nu/(1-\nu)$ The forces in the plane xy are, as presented in the second chapter, as follows:

$$u=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial y},\vartheta=\frac{\partial\phi}{\partial y}-\frac{\partial\psi}{\partial x}, \quad (1)$$

Then ϕ,ψ we get the wave equation (2.13) with complex coefficients for the displacement potentials

$$G_0\frac{2}{1-\nu}(1-i\Gamma_{\lambda\mu})\nabla^2\phi=\rho\frac{\partial^2\varphi}{\partial t^2},$$
$$G_0(1-i\Gamma_{\mu})\nabla^2\psi=\rho\frac{\partial^2\psi}{\partial t^2}. \quad (2)$$

In this case G_0, the instantaneous shear modulus, is the density of ρ the material, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the Laplace operator, $\Gamma_\mu=-i\Gamma_\mu^{(c)}(\omega_R)+\Gamma_\mu^{(s)}(\omega_R),\ \Gamma_{\lambda\mu}=-i\Gamma_{\lambda\mu}^{(c)}(\omega_R)+\Gamma_{\lambda\mu}^{(s)}(\omega_R)$. In this case, the voltage component will be as follows:

$$\sigma_x=\frac{2\bar{G}}{1-\nu}(\frac{\partial u}{\partial x}+\nu\frac{\partial\vartheta}{\partial y}),\sigma_y=\frac{2\bar{G}}{1-\nu}(\nu\frac{\partial u}{\partial x}+\frac{\partial\vartheta}{\partial y}),$$
$$\sigma_{xy}=\bar{G}(\frac{\partial\vartheta}{\partial x}+\frac{\partial u}{\partial y}). \quad (3)$$

The solution of this equation (3.2) is sought as follows

$$\phi=\Phi(x,y)e^{i\omega t},\psi=\Psi(x,y)e^{i\omega t}, \quad (4)$$

where $\Phi(x,y),\Psi(x,y)-$ are the amplitudes of the longitudinal and transverse displacement potentials ω and are the complex frequency. If we substitute the solution (4) into (2), we get the following equations

$$\nabla^2\Phi+h^2\Phi=0,\ \nabla^2\Psi+k^2\Psi=0, \quad (3.5)$$

herein

$$h^2=\frac{(1-\nu)\rho}{2G_0(1-i\Gamma_{\lambda\mu})}\omega^2,k^2=\frac{\rho}{G_0(1-i\Gamma_{\mu})}\omega^2.$$

Consider the solution of equation (3.5) above. Equations (1)-(5) are relevant for a three-layer plate. We build a structure that is symmetrical with respect to the middle plate. Let the two outer plates be identical.

$$\Phi_1 = B_1 sh\alpha_1 y \sin\xi x,\ \Psi_1 = D_1 ch\beta_1 y \cos\xi x,$$
$$\Phi_2 = [A_2 ch\alpha_2(c-y) + B_2 sh\alpha_2(c-y)]\sin\xi x,$$
$$\Psi_2 = -[C_2 sh\beta_2(c-y) + D_2 ch\beta_2(c-y)]\cos\xi x, \quad (6)$$

herein $\alpha_j^2 = \xi^2 - h_j^2,\ \beta_j^2 = \xi^2 - k_j^2.$

Based on (1), we obtain the analytical expression of displacements

$$u_1 = (B_1\xi sh\alpha_1 y + D_1\beta_1 sh\beta_1 y)\cos\xi x e^{i\omega t},$$
$$\vartheta_1 = (B_1\alpha_1 ch\alpha_1 y + D_1\xi ch\beta_1 y)\sin\xi x e^{i\omega t},$$
$$u_2 = (A_2\xi ch\alpha_2(c-y) + B_2\xi sh\alpha_2(c-y) +$$
$$+C_2\beta_2 ch\beta_2(c-y) + D_2\beta_2 sh\beta_2(c-y))\cos\xi x e^{i\omega t},$$
$$\vartheta_2 = -(A_2\alpha_2 sh\alpha_2(c-y) + B_2\alpha_2 ch\alpha_2(c-y) +$$
$$+C_2\xi sh\beta_2(c-y) + D_2\xi ch\beta_2(c-y))\sin\xi x e^{i\omega t}. \quad (7)$$

The outer surfaces of the structure under study are freed from stresses.

$$y = \pm H,\ \sigma_{y,2} = \sigma_{xy,2} = 0. \quad (8)$$

If we substitute (8) for condition (7), we get an algebraic equation

$$A_2(\xi^2 + \beta_2^2) + 2C_2\xi\beta_2 = 0,\ 2B_2\xi\beta_2 + D_2(\xi^2 + \beta_2^2) = 0. \quad (9)$$

In addition, the condition of continuity at the point of contact between the plate and the filler must be met.

$$y = \pm H_1,\ \sigma_{y,1} = \sigma_{y,2}, \sigma_{xy,1} = \sigma_{xy,2}, u_1 = u_2,\ \vartheta_1 = \vartheta_2. \quad (10)$$

If we use the above equations, we get the following system of equations for finding unknown integral constants:

$$B_1\xi sh\alpha_1 H_1 + D_1\beta_1 sh\beta_1 H_1 =$$
$$= A_2(\xi ch\alpha_2 H_2 - \frac{\xi^2 + \beta_2^2}{2\xi} ch\beta_2 H_2) +$$
$$+B_2(\xi sh\alpha_2 H_2 - \frac{2\xi\alpha_2\beta_2}{\xi^2 + \beta_2^2} sh\beta_2 H_2),$$
$$B_1\alpha_1 ch\alpha_1 H_1 + D_1\xi_1 ch\beta_1 H_1 =$$
$$= A_2(-\alpha_2 sh\alpha_2 H_2 + \frac{\xi^2 + \beta_2^2}{2\beta_2} sh\beta_2 H_2) +$$
$$+B_2(-\alpha_2 ch\alpha_2 H_2 + \frac{2\xi^2\alpha_2}{\xi^2 + \beta_2^2} ch\beta_2 H_2),$$
$$B_1(\xi^2 + \beta_1^2) sh\alpha_1 H_1 + 2D_1\xi\beta_1 sh\beta_1 H_1 =$$
$$= \frac{G_{02}}{G_{01}}[A_2(\xi^2 + \beta_2^2) ch\alpha_2 H_2 - ch\beta_2 H_2)] +$$
$$+B_2\frac{G_{02}}{G_{01}}((\xi^2 + \beta_1^2) sh\alpha_2 H_2 - \frac{4\xi^2\alpha_2\beta_2}{\xi^2 + \beta_2^2} sh\beta_2 H_2),$$
$$2B_1\xi\alpha_1 ch\alpha_1 H_1 + D_1(\xi^2 + \beta_2^2) ch\beta_1 H_1 =$$
$$= \frac{G_{02}}{G_{01}}[A_2(-2\xi\alpha_2 sh\alpha_2 H_2 + \frac{(\xi^2 + \beta_2^2)^2}{2\xi\beta_2} sh\beta_2 H_2) +$$
$$+2B_2\xi\alpha_2(ch\alpha_2 H_2 + ch\beta_2 H_2)]. \quad (11)$$

It is a system of four unknown homogeneous algebraic equations with complex coefficients

(B_1, D_1, A_2, B_2). In order for this system of equations to have a non-zero solution, its main determinant must be equal to zero. From this condition, we obtain the dispersion relation or frequency equation.

Frequency equation. The ratios given in (5) above can be written as follows

$$h_1^2 = \frac{(1-\nu_1)\rho_1}{2G_{01}(1-i\Gamma_{\lambda\mu 1})}\omega^2, k_1^2 = \frac{\rho_1}{G_{01}(1-i\Gamma_{\mu 1})}\omega^2,$$

$$\alpha_1^2 = \xi^2 - h_1^2,\ \beta_1^2 = \xi^2 - k_1^2,$$

$$h_2^2 = \frac{(1-\nu_2)\rho_2}{2G_{02}(1-i\Gamma_{\lambda\mu 2})}\omega^2, k_2^2 = \frac{\rho_2}{G_{02}(1-i\Gamma_{\mu 2})}\omega^2,$$

$$\alpha_2^2 = \xi^2 - h_2^2,\ \beta_2^2 = \xi^2 - k_2^2. \quad (12)$$

The phase velocity for a V-bending beam is:

$V = \omega / \xi$ In this case, we will introduce labeling

$$V_{s0,j}^2 = \frac{G_{0j}}{\rho_j}, V_{l0,j}^2 = \frac{2G_{0j}}{\rho_j(1-\nu_j)}, j = 1, 2. \quad (13)$$

The following expressions (13) express the instantaneous velocities of the longitudinal and transverse waves. For an infinite plate, the expressions α_j and β_j have the following form:

$$\alpha_1^2 = \xi^2\left[1-(\frac{V}{V_{l0,1}})^2\right], \alpha_2^2 = \xi^2\left[1-(\frac{V}{V_{l0,2}})^2\right],$$

$$\beta_1^2 = \xi^2\left[1-(\frac{V}{V_{s0,1}})^2\right], \beta_2^2 = \xi^2\left[1-(\frac{V}{V_{s0,2}})^2\right]. \quad (14)$$

It can be seen that α_j and β_j can be real, $V_{s0,j}, V_{l0,j}$ complex or imaginary values depending on the values of the speeds. Therefore, we will consider the following cases:

1. $V_{l0,2} > V_{s0,2} > V_{l0,1} > V_{s0,1}$,
2. $V_{l0,1} > V_{s0,1} > V_{l0,2} > V_{s0,2}$,
3. $V_{l0,2} > V_{l0,1} > V_{s0,2} > V_{s0,1}$,
4. $V_{l0,1} > V_{l0,2} > V_{s0,1} > V_{s0,2}$. (15)

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$$\alpha_2 h_2 = p, \beta_2 h_2 = mp, \alpha_1 h_1 = np, \beta_1 h_1 = qp. \quad (16)$$

If we use (12) above, we get the following ratios

$$(\xi h_2)^2 = \frac{2-(1-\nu_2)m^2}{1+\nu_2}p^2, \omega^2 = \frac{2G_{02}(1-m^2)}{\rho_2 h_2^2(1+\nu_2)}p^2 \quad (17,a)$$

and

$$(\xi h_1)^2 = \frac{2a-(1-\nu_1)q^2}{1+\nu_1}p^2, \omega^2 = \frac{2G_{01}(n^2-q^2)}{\rho_1 h_1^2(1+\nu_1)}p^2 \quad (17,b)$$

From the last equation (17) we obtain the following ratio

$$n^2 = \frac{1}{1+\nu_2}(\frac{h_1}{h_2})\left\{2-(1-\nu_2)m^2 - \frac{G_{02}\rho_1(1-\nu_1)}{G_{01}\rho_2}(1-m^2)\right\},$$

$$q^2 = \frac{1}{1+\nu_2}(\frac{h_1}{h_2})\left\{2-(1-\nu_2)m^2 - 2\frac{G_{02}\rho_1}{G_{01}\rho_2}(1-m^2)\right\}.$$

If we simplify equation (11) and make the principal determinant equal to zero, we get the frequency equation

$$\begin{aligned}
&\frac{G_{01}}{G_{02}}\left\{b_1(1-\frac{1}{chpchmp})+e_1thp\,thmp\right\}\cdot \\
&\cdot(-n'q'thqp+\xi'^2thnp)+ \\
&+\frac{G_{01}}{G_{02}}\left\{\frac{b_1}{chpchmp}+e_2+f_2thp\,thmp\right\}\cdot \\
&\cdot(-4\xi'^2n'q'thqp+(\xi'^2+q'^2)^2thnp)+ \\
&+(b_1thmp+e_2thp)(\xi'^2-q'^2)q'thqp\,thnp+ \\
&+\left[b_1(1-\frac{1}{chp\,chmp})+e_4thp\,thmp\right]\cdot \\
&\cdot(2n'q'thqp-(\xi'^2+q'^2)^2thnp)+ \\
&+(b_1thp+e_5thmp)(\xi'^2-q'^2)n'=0. \qquad (18)
\end{aligned}$$

It includes the following definitions:

$$\begin{aligned}
&a=n'(\frac{h_1}{h_2}), q=q'(\frac{h_1}{h_2}),\ \xi'^2=\frac{1}{1+\nu_2}(2-(1-\nu_2)m^2), \\
&n'^2=\frac{1}{1+\nu_2}(2-(1-\nu_2)m^2-\frac{G_{20}\rho_1}{G_{10}\rho_2}(1-\nu_1)(1-m^2)), \\
&q'^2=\frac{1}{1+\nu_2}(2-(1-\nu_2)m^2-2\frac{G_{20}\rho_1}{G_{10}\rho_2}(1-m^2)), \\
&b_1=8m^2\xi'^2(m^2+\xi'^2)^2, e_1=-(m(m^2+\xi'^2)^4+m^3), \\
&b_2=-4m^2\xi'^2(m^2+\xi'^2)^2, e_2=m^2(m^2+\xi'^2)^2+4m, \\
&f_2=-\left[m\xi'^2(m^2+\xi'^2)^2+4m^3\xi'^2\right], \\
&b_3=m(-m^2+\xi'^2)(m^2+\xi'^2)^2, e_3=-4m^2\xi'^2(-m^2+\xi'^2)^2, \\
&b_4=4m^2\xi'^2(m^2+3\xi'^2)(m^2+\xi'^2)^2, \\
&e_4=-2m\xi'^2(m^2+\xi'^2)^3-16m^3\xi'^4, \\
&b_5=m^2\xi'^2(-m^2+\xi'^2)(m^2+\xi'^2)^2, e_4=-4m\xi'^2(m^2+\xi'^2)^3. \qquad (19)
\end{aligned}$$

The above transcendental complex parametric equation (18) is calculated. This equation is solved by the methods of Müller and Gauss. Numerical results are presented in Figures 1, 2 and 3.

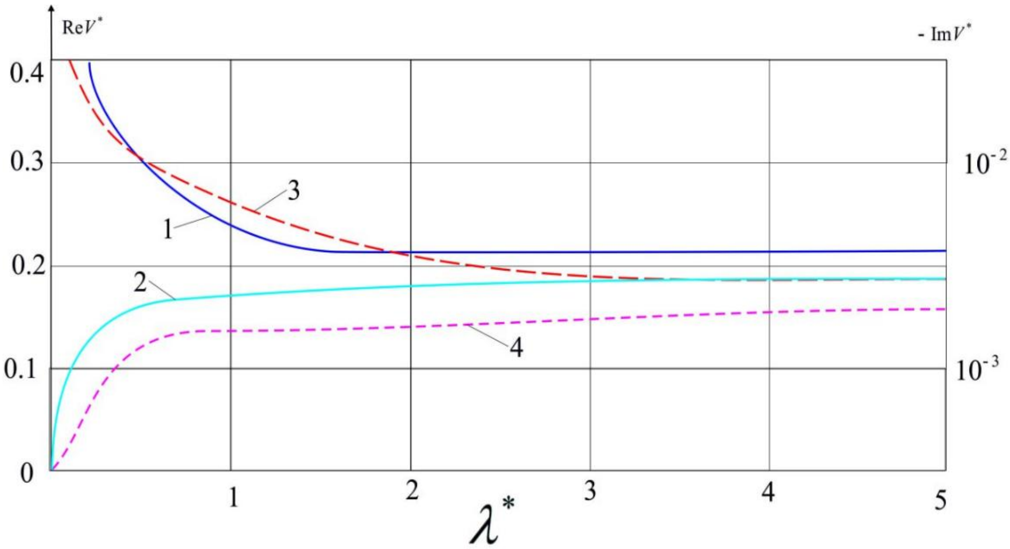

Figure 1. Change of Phase Velocity in a Three-Layer Beam as a Function of Wavelength

The numerical results show a change in the real and imaginary parts of the phase velocity in a three-layer beam with a wavelength at h2/h1 = 1/10. It can be seen that a decrease in the wavelength of the tube ($\lambda^{*} = \frac{h_2}{\lambda}$) leads to an asymptotic change in the phase velocity. The results are presented for two modes of phase velocity. The ordinate axis $\mathrm{Re}V = \mathrm{Re}\frac{V}{V_{s0,1}}$ and $\mathrm{Im}V = -\mathrm{Im}\frac{V}{V_{s0,1}}$ - (1,2 is the real part of the group velocity, 3,4 are the corresponding imaginary parts). The dependence on the number of waves is shown in Figure 2. From the figure it can be seen that at small values of $\mathrm{Re}V_g = \mathrm{Re}\frac{V_g}{V_{s0,1}}$ the velocity of wave propagation in the tube, the group phase velocity is expressed by nonmonotonic functions (1.2 is the real part of the group velocity, 3.4 is the corresponding imaginary parts).

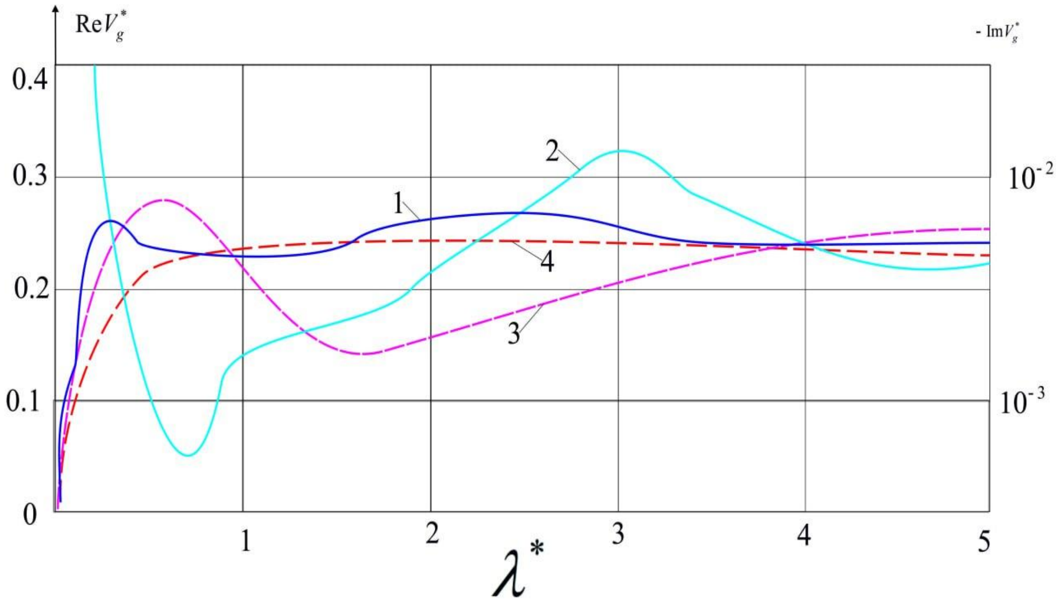

Figure 2. The change in the group velocity as a function of the wavelength in a three-layer beam is described.

In obtaining numerical results, the material of the first plate was adopted as aluminum, and the second as plastic. Their characteristics are as follows:

$G_A = 2700 kg / mm^2, \nu_A = 0.34, \rho_A g = 2.7 \cdot 10^{-3};$

$G_П = 110 kg / mm^2, \nu_A = 0.36, \rho_П g = 1.3 \cdot 10^{-3}.$

Thus, the dispersion equation for the problem of wave propagation on a three-beam hammer was derived and numerically solved. The change in group and phase velocities depending on the wavelength is analyzed.

REFERENCES

Паршина Л. В., Рябов В. М., Ярцев Б. А. Рассеяние энергии при колебаниях неоднородных композитных структур. 1. Постановка задачи // Вестник С.-Петерб. ун-та. Математика. Механика. Астрономия. 2018. Т. 5 (63). Вып. 2. С. 300–309. https://doi.org/10.21638/11701/spbu01.2018.210

Паршина Л. В., Рябов В. М., Ярцев Б. А. Рассеяние энергии при колебаниях неоднород ных композитных структур. 2. Метод решения // Вестник С.-Петерб. ун-та. Математика. Механи ка. Астрономия. 2018. Т. 5 (63). Вып. 4. С. 678–688. https://doi.org/10.21638/11701/spbu01.2018.414

Паршина Л. В., Рябов В. М., Ярцев Б. А. Рассеяние энергии при колеба ниях неоднородных композитных структур. 3. Численный эксперимент // Вестник С. Петерб. ун-та. Математика. Механика. Астрономия. 2019. Т. 6 (64). Вып. 1. С. 144–156. https://doi.org/10.21638/11701/spbu01.2019.111

Berthelot J.-M. Damping analysis of orthotropic composites with interleaved viscoelas tic layers: modeling // Journal of Composite Materials. 2006. Vol. 40. Iss.21. P. 1889–1909. https://doi.org/10.1177/0021998306061302

Berthelot J.-M., Sefrani Y. Damping analysis of unidirectional glass fiber composites with inter leaved viscoelastic layers: experimental investigation and discussion // Journal of Composite Materials. 2006. Vol. 40. Iss.21. P. 1911–1932. https://doi.org/10.1177/0021998306061303

Hao M., Rao M.D. Vibration and Damping Analysis of a sandwich beam containing a vis coelastic constraining layer // Journal of Composite Materials. 2005. Vol. 39. Iss.18. P. 1621–1643. https://doi.org/10.1177/0021998305051124