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# FIXED POINTS OF LINEAR OPERATORS WHICH MAP OF SIMLEX TO ITSELF IN THE CASE FOR n=3

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## ANNOTATION

Identpotent mathematics consists of changing simple arithmetic operations for a new set of main operations (as maximum or minimum), in this case a field of numbers exchange with idenpotent semirings and semifields.

## INTRODUCTION

As typical examples we can take algebras Maks-plus $R_{\max}$ and min-plus $R_{\min}$.

Let $R-$ be a field of real numbers. Then The operations in $R_{\max}=R\cup\{-\infty\}$ are in the following: $x\oplus y=\max\{x,y\}$ and $x\otimes y=x+y$.

Similarly, The operations in $R_{\min}=R\cup\{+\infty\}$ are in the following:

$\oplus=\min,\ \otimes=+$

We consider with $R_{\max}=R\cup\{-\infty\}$ idempotent addition and multiplication operations .

We define a simlex of idempotent measures with $(n-1)-$ dimension in the following case:

$$I_n=\left\{(x_1,\ldots x_n)\in R^n_{\max}:\max_{1\le i\le n} x_i=0\right\}=$$
$$=\left\{(x_1,\ldots x_n)\in R^n_{\max}:x_1\oplus\ldots\oplus x_n=1\right\}.$$

The following theorem show the form of the operations which maps of $I_n$ to itself.

We give this theorem without the proof.

<u>**Theorem1.**</u> $A=(a_{ij})_{i,j=\overline{1,n}}$ linear operator $I_n$ It is necessary and sufficient to satisfy one of the following conditions for self-reflection:

1) $a_{ij}\ge 0$ and $A$ the matrix has at least 1 zero line;
2) $a_{ij}\ge 0$ and $A$ all rows and columns of the matrix contain no more than one non-zero element. [10]

Suppose that condition 2) of this theorem is satisfied.

that is $a_{ij}\ge 0$ and $A$ all rows and columns of the matrix contain no more than one non-zero element

$$A=\begin{pmatrix}0 & 0 & a_{13}\\ a_{21} & 0 & 0\\ 0 & a_{32} & 0\end{pmatrix} \text{ case}$$

Using the method of mathematical induction $A^n$ find the general view of the matrix,

$$A\times A=\begin{pmatrix}0 & 0 & a_{13}\\ a_{21} & 0 & 0\\ 0 & a_{32} & 0\end{pmatrix}\begin{pmatrix}0 & 0 & a_{13}\\ a_{21} & 0 & 0\\ 0 & a_{32} & 0\end{pmatrix}=\begin{pmatrix}0 & a_{13}a_{32} & 0\\ 0 & 0 & a_{21}a_{13}\\ a_{32}a_{21} & 0 & 0\end{pmatrix},$$

$$A\times A\times A=\begin{pmatrix}0 & a_{13}a_{32} & 0\\ 0 & 0 & a_{21}a_{13}\\ a_{32}a_{21} & 0 & 0\end{pmatrix}\begin{pmatrix}0 & 0 & a_{13}\\ a_{21} & 0 & 0\\ 0 & a_{32} & 0\end{pmatrix}=\begin{pmatrix}a_{21}a_{13}a_{32} & 0 & 0\\ 0 & a_{21}a_{13}a_{32} & 0\\ 0 & 0 & a_{21}a_{13}a_{32}\end{pmatrix},$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$1)\ A^{3k-2}=\begin{pmatrix}0 & 0 & a_{21}^{k-1}a_{13}^{k}a_{32}^{k-1}\\ a_{21}^{k}a_{13}^{k-1}a_{32}^{k-1} & 0 & 0\\ 0 & a_{21}^{k-1}a_{13}^{k-1}a_{32}^{k} & 0\end{pmatrix}$$

$$2)\ A^{3k-1}=\begin{pmatrix}0 & a_{21}^{k-1}a_{13}^{k}a_{32}^{k} & 0\\ 0 & 0 & a_{21}^{k}a_{13}^{k}a_{32}^{k-1}\\ a_{21}^{k}a_{13}^{k-1}a_{32}^{k} & 0 & 0\end{pmatrix}$$

$$3)\ A^{3k}=\begin{pmatrix}a_{21}^{k}a_{13}^{k}a_{32}^{k} & 0 & 0\\ 0 & a_{21}^{k}a_{13}^{k}a_{32}^{k} & 0\\ 0 & 0 & a_{21}^{k}a_{13}^{k}a_{32}^{k}\end{pmatrix}$$

Our operator will appear. In that case, let's look at the above three cases.

**1 case**. n=3k-2 let it be.

$$A^n x^0=\begin{pmatrix}0 & 0 & a_{21}^{k-1}a_{13}^{k}a_{32}^{k-1}\\ a_{21}^{k}a_{13}^{k-1}a_{32}^{k-1} & 0 & 0\\ 0 & a_{21}^{k-1}a_{13}^{k-1}a_{32}^{k} & 0\end{pmatrix}\begin{pmatrix}x_1^0\\ x_2^0\\ x_3^0\end{pmatrix}=\begin{pmatrix}a_{21}^{k-1}a_{13}^{k}a_{32}^{k-1}x_3^0\\ a_{21}^{k}a_{13}^{k-1}a_{32}^{k-1}x_1^0\\ a_{21}^{k-1}a_{13}^{k-1}a_{32}^{k}x_2^0\end{pmatrix} \text{ is equal to.}$$

Thus, it is optional $x^0=(x_1^0,\ x_2^0,\ x_3^0)\in I_3$ for

$x^{(n)}=A^n x^0=(a_{21}^{k-1}a_{13}^{k}a_{32}^{k-1}x_3^0, a_{21}^{k}a_{13}^{k-1}a_{32}^{k-1}x_1^0, a_{21}^{k-1}a_{13}^{k-1}a_{32}^{k}x_2^0)$ we have.

**2 cases** n=3k-1 let it be.

$$A^n x^0=\begin{pmatrix}0 & a_{21}^{k-1}a_{13}^{k}a_{32}^{k} & 0\\ 0 & 0 & a_{21}^{k}a_{13}^{k}a_{32}^{k-1}\\ a_{21}^{k}a_{13}^{k-1}a_{32}^{k} & 0 & 0\end{pmatrix}\begin{pmatrix}x_1^0\\ x_2^0\\ x_3^0\end{pmatrix}=\begin{pmatrix}a_{21}^{k-1}a_{13}^{k}a_{32}^{k}x_2^0\\ a_{21}^{k}a_{13}^{k}a_{32}^{k-1}x_3^0\\ a_{21}^{k}a_{13}^{k-1}a_{32}^{k}x_1^0\end{pmatrix} \text{ is equal to}$$

Thus, it is optional $x^0=(x_1^0,\ x_2^0,\ x_3^0)\in I_3$ for

$x^{(n)}=A^n x^0=(a_{21}^{k-1}a_{13}^{k}a_{32}^{k}x_2^0, a_{21}^{k}a_{13}^{k}a_{32}^{k-1}x_3^0, a_{21}^{k}a_{13}^{k-1}a_{32}^{k}x_1^0)$ we have

**3 cases** n=3k let it be

$$A^n x^0 = \begin{pmatrix} a_{21}^k a_{13}^k a_{32}^k & 0 & 0 \\ 0 & a_{21}^k a_{13}^k a_{32}^k & 0 \\ 0 & 0 & a_{21}^k a_{13}^k a_{32}^k \end{pmatrix} \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \end{pmatrix} = \begin{pmatrix} a_{21}^k a_{13}^k a_{32}^k x_1^0 \\ a_{21}^k a_{13}^k a_{32}^k x_2^0 \\ a_{21}^k a_{13}^k a_{32}^k x_3^0 \end{pmatrix}$$ is equal to

Thus, it is optional $x^0 = (x_1^0,\ x_2^0,\ x_3^0) \in I_3$ for

$x^{(n)} = A^n x^0 = (a_{21}^k a_{13}^k a_{32}^k x_1^0, a_{21}^k a_{13}^k a_{32}^k x_2^0, a_{21}^k a_{13}^k a_{32}^k x_3^0)$ we have

$n \to \infty$ da, $\Rightarrow$ $k \to \infty$ Obviously, we calculate the following limit.

**Lemma3:** An A-line operator satisfies the second condition of the theorem $a_{13} \geq 0$, $a_{21} \geq 0$ $a_{32} \geq 0$ the rest $a_{ij} = 0$ if so $I_3$ forms trajectories as follows

**3.1.** n=3k-2 let it be

$$\lim_{n\to\infty} x^{(n)} = \lim_{k\to\infty}(a_{21}^{k-1} a_{13}^k a_{32}^{k-1} x_3^0, a_{21}^k a_{13}^{k-1} a_{32}^{k-1} x_1^0, a_{21}^{k-1} a_{13}^{k-1} a_{32}^k x_2^0) =$$

$$= \lim_{k\to\infty}((a_{13}a_{21}a_{32})^{k-1} a_{13}x_3^0, (a_{13}a_{21}a_{32})^{k-1} a_{21}x_1^0, (a_{13}a_{21}a_{32})^{k-1} a_{32}x_2^0) =$$

$$= \begin{cases} (0,0,0), & agar \quad a_{13}a_{21}a_{32} < 1 \quad bo'lsa, \\ \left(a_{13}x_3^0, a_{21}x_1^0, a_{32}x_2^0\right) & agar \quad a_{13}a_{21}a_{32} = 1 \quad bo'lsa, \\ (-\infty,-\infty,-\infty), & agar \quad a_{13}a_{21}a_{32} > 1 \quad bo'lsa. \end{cases}$$

**3.2.** n=3k-1 let it be

$$\lim_{n\to\infty} x^{(n)} = \lim_{k\to\infty}(a_{21}^{k-1} a_{13}^k a_{32}^k x_2^0, a_{21}^{\mathrm{k}} a_{13}^k a_{32}^{k-1} x_3^0, a_{21}^k a_{13}^{k-1} a_{32}^k x_1^0) =$$

$$= \lim_{k\to\infty}((a_{13}a_{21}a_{32})^{k-1} a_{32}a_{13}x_2^0, (a_{13}a_{21}a_{32})^{k-1} a_{13}a_{21}x_3^0, (a_{13}a_{21}a_{32})^{k-1} a_{21}a_{32}x_1^0) =$$

$$= \begin{cases} (0,0,0), & agar \quad a_{13}a_{21}a_{32} < 1 \quad bo'lsa, \\ \left(a_{13}a_{32}x_2^0, a_{21}a_{13}x_3^0, a_{32}a_{21}x_2^0\right) & agar \quad a_{13}a_{21}a_{32} = 1 \quad bo'lsa, \\ (-\infty,-\infty,-\infty), & agar \quad a_{13}a_{21}a_{32} > 1 \quad bo'lsa. \end{cases}$$

**3.3.** n=3k let it be

$$\lim_{n\to\infty} x^{(n)} = \lim_{k\to\infty}(a_{21}^k a_{13}^k a_{32}^k x_1^0, a_{21}^k a_{13}^k a_{32}^k x_2^0, a_{21}^k a_{13}^k a_{32}^k x_3^0) =$$

$$= \lim_{k\to\infty}((a_{13}a_{21}a_{32})^k x_1^0, (a_{13}a_{21}a_{32})^k x_2^0, (a_{13}a_{21}a_{32})^k x_2^0) =$$

$$= \begin{cases} (0,0,0), & agar \quad a_{13}a_{21}a_{32} < 1 \quad bo'lsa, \\ \left(x_1^0, x_2^0, x_3^0\right) & agar \quad a_{13}a_{21}a_{32} = 1 \quad bo'lsa, \\ (-\infty,-\infty,-\infty), & agar \quad a_{13}a_{21}a_{32} > 1 \quad bo'lsa. \end{cases}$$

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