GALAXY INTERNATIONAL INTERDISCIPLINARY RESEARCH JOURNAL (GIIRJ)
ISSN (E): 2347-6915
Vol. 12, Issue 5 May (2024)

APPLYING THE PRINCIPLE OF LEAST EFFECT IN ECONOMICS

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ABSTRACT

This paper examines questions about the possibility of using methodological approaches of natural and mathematical sciences to the organization and study of economic processes. In particular, the expediency of conducting economic research based on the principle of least action is noted

Keywords: space of the economic system, product, asset, good, phase space, trajectory, principle of least action, calculus of variations, functional, Lagrange function or Lagrangian, Euler–Lagrange equations.

INTRODUCTION

Mathematical models of the vast majority of closed physical systems are based on variational principles, i.e. it is postulated that the equations describing the evolution of the system are Euler–Lagrange equations of some functional. In this regard, variational methods are one of the main tools for studying numerous problems in natural science.

It is known that problems that allow variational formulation make it possible to maximally weaken the mathematical restrictions imposed on the solutions being sought, as well as to construct a priori stable difference schemes for their numerical implementation. The calculus of variations lies at the origins of the theory of optimal control and optimal design of structures. That is why variational methods are so popular in mechanics, physics and engineering calculations.

The calculus of variations also finds numerous applications in economics (eg [1], [2], [8]).

In this paper, a variational approach is proposed to describe the economic system, which, without questioning the basic economic laws, will allow us to look at them from a different angle.

RESEARCH METHODOLOGY

The variational method was used - one of the most powerful tools for obtaining equations of motion of physical, chemical and other systems (the principle of least action), used in the natural sciences .

LITERATURE REVIEW

The principle of least action was first proposed by the French mathematician and mechanic Pierre Louis Moreau de Maupertuis. This principle was formulated in a vague form and without proof in the work “The reconciliation of various laws of nature, which until now seemed incompatible” [3]. Maupertuis formulates his principle, according to which the true

trajectory of a particle differs from any other in that the action for it is minimal (Maupertuis principle) [4]. Having proclaimed a new law of nature, which consists in the minimality of action, Maupertuis did not, however, give a clear definition of the value that needs to be minimized [5]. Maupertuis proclaimed the principle of least action to be the most general law of nature. Maupertuis justified the universality of the principle of least action on the basis of metaphysical ideas about nature, where everything should occur from some reasonable considerations, as if nature in its actions pursues some reasonable goals that it sets for itself.
Mathematicians: Euler, Lagrange, Hamilton, Ostrogradsky, Jacobi - established mathematically rigorous expressions for the principle of least action. The first mathematical formulation of the principle of least action was given by Euler. According to Lagrange, it sounds like this: "For trajectories described by bodies under the action of central forces, the integral of the velocity multiplied by the element of the curve will always be a maximum or minimum" [5]. In 1834-1835, William Rowan Hamilton published [5] a new variational principle, now known as the principle of stationary action or Hamilton's principle. The principle of least action was admired and dedicated to it in separate scientific works by L. Euler, J. Lagrange, J. D'Alembert, W. Hamilton, K. Gauss, G. Hertz, G. Helmholtz, A. Poincaré, A. Einstein, E. Schrödinger, M. Planck, R. Feynman and many others.
The principle of least action in economics is discussed in the works of I.G. Tsareva [6] and [7], etc.

RESEARCH AND RESULTS

1 . Space of the economic system . Every economic event must occur in a specific space. This means that the first step in creating any model must be to define the space in which the system is defined. A certain coordinate reference system (economic variables) is associated with space , allowing one to determine the position of any point relative to the origin of coordinates (reference point).
We also assume that the chosen coordinates are significant, i.e. completely define the economic system (they completely describe the important features of the system for us).
It is advisable to select economic variables in such a way that they are comparable with each other, i.e. so that a common unit of measurement can be established for them. In this case, the space is metric, i.e. you can introduce the concept of distance and direction, for example from the origin to a specific point.
As possible economic variables, we highlight the following categories: goods (services), assets and benefits. A commodity is a product of labor intended for exchange through purchase and sale. The ability to exchange for other goods is given to a commodity by value - abstract labor. True, air, undeveloped lands, natural meadows, wild forests were not created by labor, although as useful things they have their price. In contrast to a product, an asset is any value owned by a participant in an economic system. It seems that this economic category suits us better than the product, although the category product (service) can also be used with the mentioned reservations. The category good - any consumable object that brings some satisfaction to the consumer and therefore has its own price - is used together with an asset. Let us note that, when necessary, it is necessary to distinguish between two large groups:

consumer goods that satisfy people's needs directly, and production goods that satisfy people's needs indirectly.

The simplest way is to define as the coordinates $\left(x_i, i=1,2,...,n\right)$ of our space $\mathbf{R}^n$ the quantities of goods (assets, goods) of our system, expressed in some conventional units. The position of a point in space specifies the corresponding quantities of assets existing in the system at a given point in time. Obviously, these coordinates will only take positive values. The position of a point is determined by the direction towards it from the origin and the distance from it, i.e. characterized by a state vector. Let us assume that the coordinates clearly depend only on time and are independent of each other.

It is clear that if the system has a certain set of goods, then the quantities of these goods are components of the radius vector that determines the position of the point depicting the position of the economic system in our space. In the case when the quantities of goods are unchanged, then the point is motionless (the system is at rest), but this state is not interesting for analysis. When the quantities of goods begin to change, the system begins to move. Then we must take into account the speed and direction of motion of the system, introducing additional coordinates for this purpose $\left(x_i', i=1,2,...,n\right)$, the amount of goods (assets, goods) of our system that are produced and consumed in the system over a certain period of time (unit of time). This means that we introduce as additional coordinates derivatives of the amount of goods over time, which give the rate of change of goods in the system. In the future, unless otherwise specified, when we talk about the quantity of any good that was produced (output) or consumed, we will understand this quantity as produced or consumed over a certain time. A significant difference from simple quantities of goods, which can only take positive values, our additional coordinates can take on any real values. It should be noted that there is some confusion in the economic literature on this issue. This confusion lies in the fact that very often they speak of output as the quantity of a good and designate this quantity as x, forgetting to mention that they mean the quantity of a good produced per unit of time, i.e. speed x'. It is well known that the simultaneous specification of all coordinates and velocities completely determines the state of the system, i.e. defines a point in space, called a representing point, and allows, in principle, to predict its further movement. Thus, it is not necessary to specify any derivatives of higher (than zero and first) order, for example x_i'', x_i''', to fully describe the motion of the system. This non-trivial fact is a law of nature. The space $\mathbf{R}^{2n}$ defined in this way is called phase space.

Many economic phenomena and not only they are described using the function f. For example, if at each point in space or some of its sets the value of a certain quantity is defined, then they say that a function or field of this quantity is given, depending on several variables n of its coordinates x_i:, $f\left(x_i\right)$ which, in turn, are functions of time t. Depending on the nature of the given quantity, one speaks of a scalar or vector field in the considered set (region) of space. The movement of an economic system (its representing point) in economic space is described by a trajectory. A trajectory is the geometric location of points where an economic system has visited during its movement, a function of its coordinates x_i, which, in turn, are functions of time t. It is not permissible to confuse a trajectory with an integral curve described by a point (t, x_i) in space $\mathbf{R}^{2n+1}$. As the economic system moves, its functions also change. We will assume that the functions we encounter are smooth, i.e. continuously differentiable the required number of times.

Functions are determined from equations that express mathematically the laws that govern the phenomenon under study. In most cases, these equations contain derivatives of the desired functions. Such equations are called differential equations.

2. Principle of least action . Calculus of variations is a branch functional analysis, which deals with finding extrema (minimum and maximum) of functionals, i.e. functions whose domain of definition is an infinite-dimensional space, whose elements are curves.

If we consider the movement of an arbitrary system from one point in space to another, then, generally speaking, the system can make this transition along different trajectories, each of which represents a certain curve. A possible set of trajectories is given by the equations of the dynamic system and can, generally speaking, be infinite. However, in most cases, except for some exceptional ones (i.e. under "normal conditions"), the system will move in such a way that the length of this curve is the minimum possible. This principle of motion of a dynamic system, which is also a law of nature, is called "Hamilton's principle of least action." This variational principle allows us to consider a series of successive states of the system over a finite period of time or, what is the same, on a finite segment of the trajectory and compare them with neighboring virtual states that are in a certain correspondence with them. From the point of view of the principle of least action, nature achieves its goal in the most direct way, therefore, with the least expenditure of funds. Therefore, it would be more appropriate to use the term "the principle of least cost with greatest effect." But the term "action", introduced and used by Helmholtz and Planck, has firmly entered into everyday use, and any replacement of it with another term would be unpromising.

In a historically earlier interpretation, the principle of least action was formulated in the form of the so-called Maupertuis principle, but we will focus on Hamilton's principle of least action. Since the length of the trajectory from point (0) to point (1) is a certain function F of coordinates x_i, x_i' which, in turn, are functions of time t, we can write:

$$F(x_i) = \int_{t_0}^{t_1} f(x_i(t), x_i'(t), t)dt,$$

where F is called a functional f or action, and a function f called the Lagrange function or Lagrangian is such that the action F has the minimum value of all possible ones. This minimum value of the functional F is called its extremal. The fact that the Lagrange function contains only x_i and x_i', but not higher derivatives, is an expression of the above statement that the state of the system is completely determined by specifying coordinates and velocities.

It is clear that the value of the functional changes when moving from an extremal trajectory to another, fairly close one. This close trajectory can be obtained by variation. Let us consider the increment of the functional δΦwhen the coordinates change by δx_i and $\delta x_i' = \frac{d}{dt}\delta x_i$, called its variation or differential, where δx_i are arbitrary differentiable functions that take small values, and all $\delta x_i(t_0)=\delta x_i(t_1)=0$, i.e. when varying the extremal, the starting and ending points remain fixed. A necessary condition for a function to be extremal is for the set of terms of its differential to vanish.

$$\delta F=\int_{t_0}^{t_1} f(x_i(t),x_i'(t),t)dt=\sum_i\int_{t_0}^{t_1}\left(\frac{\partial f}{\partial x_i}\delta x_i+\frac{\partial f}{\partial x_i'}\delta x_i'\right)dt=\sum_i\int_{t_0}^{t_1}\left(\frac{\partial f}{\partial x_i}\delta x_i+\frac{\partial f}{\partial x_i'}\frac{d}{dt}x_i\right)dt=0.$$

Integrating by parts the second integrand

$$\frac{\partial f}{\partial x_i'}\frac{d}{dt}\delta x_i=\frac{d}{dt}\left(\frac{\partial f}{\partial x_i'}\delta x_i\right)-\delta x_i\frac{\partial f}{\partial x_i'},$$

we get:

$$\delta F=\sum_i\frac{\partial f}{\partial x_i'}\delta x_i\Bigg|_{t_0}^{t_1}+\sum_i\int_{t_0}^{t_1}\left(\frac{\partial f}{\partial x_i}-\frac{d}{dt}\left(\frac{\partial f}{\partial x_i'}\right)\right)\delta x_i dt=0.$$

The first term is equal to zero due to the setting of boundary conditions. What remains is the integral, which must be equal to zero for arbitrary and independent values of δx_i. This is only possible if the integrand vanishes identically (Dubois-Reymond Lemma) .

Thus we get the equations

$$\frac{d}{dt}\left(\frac{\partial f}{\partial x_i'}\right)-\frac{\partial f}{\partial x_i}=0.$$

These differential equations are called the Euler–Lagrange equations for the functional F. They establish a connection between accelerations, velocities and coordinates, i.e. represent the equations of motion of a system, regardless of its type.

From a mathematical point of view, the Euler–Lagrange equations constitute a system of n second-order equations for n unknown functions. The general solution of such a system contains $2n$ arbitrary constants. To determine them, knowledge of the initial conditions is necessary, for example, knowledge of the initial values of all coordinates and velocities.

3. The economic meaning of the principle of least action. To explain the economic meaning of the functional (action), the Lagrange function and the Euler–Lagrange equations, let us consider in more detail the concept of an economic system. It is known that in economic theory there are different definitions of an economic system:

– as a specific converter of the “nature-society” flow, regulating the way of converting natural resources into people’s means of subsistence;

– as a social form of expression of a technological method of connecting factors of production;

– as a connecting link between the “nature” system and the “society” system.

An essential property of an economic system is the presence of two poles of social reproduction – “input” and “output”. Natural and labor resources are involved $(x_i'<0)$ in the “input” – the process of asset production – and the “output” – the process of asset consumption $(x_i'>0)$– involves the assets themselves – consumer goods. Assets may or may not participate in the production process. In this sense, assets as goods (i.e., consumer goods that bring a certain satisfaction to the consumer) are divided into consumer goods - satisfying people's needs directly, and production goods - satisfying people's needs indirectly. Between these two poles there is a process of implementation - the distribution and exchange of produced goods, which we will consider later.

Let us define the first term of the Euler–Lagrange equations $\frac{d}{dt}\left(\frac{\partial f}{\partial x_i'}\right)$ as responsible for the "production pole", and the second term $\frac{\partial f}{\partial x_i}$ as responsible for the "consumption pole". Moreover, the magnitude

$$p_i' = \frac{d}{dt}\left(\frac{\partial f}{\partial x_i'}\right) = \frac{\partial f}{\partial x_i},$$

where p_i is the price, in the first case we define it as the first derivative with respect to time of the supply price, and in the second - as the first derivative with respect to time of the demand price.

The offer price is the minimum price at which the seller agrees to sell a certain amount of a good (asset, good or service) that he has produced over a certain period (unit) of time.

The demand price is the maximum price at which a buyer intends and is able to purchase a certain amount of a good over a certain period of time.

In this case, the Lagrange function is, generally speaking, the difference between the aggregate supply and aggregate demand functions multiplied by the equilibrium price.

Aggregate demand is the total amount of goods that can be demanded (or purchased per unit of time) at various price levels.

Aggregate supply is the total quantity of goods that can be supplied (produced per unit of time) at different price levels.

Equilibrium price is the price in a competitive market at which the quantity demanded and the quantity supplied are equal.

Thus, the principle of least action in an economic system reflects the general economic equilibrium, i.e. a situation in which equality between supply and demand is simultaneously achieved in all markets. As a result, the transition of the system from one state to another occurs along the shortest path, i.e. supply and demand are growing as quickly as possible.

In the process of development (movement) of an economic system, not all produced goods are consumed, some can be accumulated. First of all, this applies to those goods that can be used repeatedly (liquid assets - cash, gold, deposits; durable goods; real estate; land, etc.), but goods that are used for consumption only once can also be accumulated times, if they have a sufficient shelf life without loss of their consumer properties. The totality of these accumulated assets owned by the subjects of the economic system forms wealth . The total stock of wealth of a country is determined by the indicator of the so-called "realizable property", which refers to physical and financial assets characterized by a sufficiently high level of liquidity (quick marketability). The amount of wealth accumulated over time is calculated as a functional (action). With the most rapid development of an economic system, when supply and demand grow at maximum speed, the wealth of the system takes on the minimum value of all possible values (it is an extreme of the functional).

The introduction of the concept of price is also important from the following point of view. Considering the prices of goods as coordinates, we can replace the coordinates in our system. We can move from coordinates (x_i', x_i) to coordinates (x_i', p_i) or coordinates (x_i, p_i), which will also be independent and essential. Which coordinate system to use in the future will be determined only by considerations of convenience.

As mentioned above, the principle of least action specifies the motion of the system under “normal conditions”, i.e. for most cases. With such movement, the system passes through a continuous sequence of equilibrium states and moves along the shortest path. If a system is brought out of equilibrium, it deviates from its extremal, and the path of the system lengthens.

conclusions

Application of the principle of least action to solving practical problems of economics can significantly increase the efficiency of economic processes. This conclusion follows from the objectively existing fact of the material basis of the phenomena and processes of the world around us. All these phenomena and processes cannot but correspond and obey its fundamental laws. Economic processes and phenomena must correspond and obey these same laws. And just as the principle of least action plays a leading role in natural processes and phenomena, so this principle must play a leading role in economic processes.

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