APPLICATION OF UNCERTAIN JERK CHAOTIC SYSTEMS IN SECURE COMMUNICATION SYSTEM

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ABSTRACT

This paper first proposes a global exponential state estimator for uncertain jerk chaotic systems. Then, based on the above state estimators, a chaotic secure communication system with multiple security effects is planned to be designed. The transmission signal and the recovery signal of this secure communication system will be synchronized in an exponential convergence manner, and the exponential convergence rate is also derived. Finally, multiple numerical simulation results are supplemented to verify and illustrate the correctness of the main theorem.

Keywords: uncertain systems, state estimator, secure communication system, chaotic systems

INTRODUCTION

Physical systems contain more or less uncertain factors; such as unknown system parameters, incomplete models or unknown external noise. The above factors often make the design of state estimators for uncertain systems more difficult, and have become a major issue that scholars urgently want to solve. On the other hand, since chaotic systems are nonlinear systems and have signal unpredictability, this makes the design of state estimators for chaotic systems more challenging. Based on the above motivations, the primary topic of this paper is to try to construct a state estimator for uncertain jerk chaotic systems [1].

In recent years, various studies on secure communication systems have continued to receive attention, exploration and discussion; see, for example, [2]-[20] and the references therein. As we know, a good communication system must not only be able to achieve synchronization in receiving and transmitting signals, but also take into account the confidentiality of the entire system's transmission.

Taking a look at the above reasons, we want to use the state estimator of the uncertain jerk chaotic systems as a basis to design a chaotic secure communication system with multiple security functions at the same time. In addition to rigorous theoretical support, this communication system is supplemented by multiple numerical simulation results to verify and illustrate the correctness of this secure communication system. Throughout this paper, $\|x\| := \sqrt{x^T \cdot x}$ denotes the Euclidean norm of the column vector x and $|a|$ denotes the absolute value of a real number a.

PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we consider the following uncertain jerk chaotic systems

$$\dot{x}_1(t) = x_2(t), \tag{1a}$$

$$\dot{x}_2(t) = \Delta q_1 x_2(t) + \Delta q_2 x_1^3(t) + \Delta q_3 \cos(wt), + \Delta f(x_1(t), x_2(t)), \tag{1b}$$

$$y(t) = \lambda_1 x_1(t) + \lambda_2 x_2(t), \ \forall t \ge 0, \tag{1c}$$

where $x(t) := [x_1(t) \quad x_2(t)]^T \in \Re^{2\times 1}$ is the state vector, $y(t) \in \Re$ is the system output, $\Delta q_1, \Delta q_2$, and Δq_3 are uncertain parameters of state equation, $\Delta f(x_1(t), x_2(t))$ are unknown external excitations, λ_1 and λ_2 are parameters of output equation with $\lambda_1 \cdot \lambda_2 > 0$. Undoubtedly, the state variables of uncertain systems are difficult to estimate or measure for various reasons. Therefore, the development of state estimators for uncertain systems (1) is currently a major challenge for scholars.

Remark 1. It is noted that the traditional jerk chaotic system [21] is the special case of systems (1) with $\Delta q_1 = -0.06$, $\Delta q_2 = -1$, $\Delta q_3 = 5$, $w = 1$, and $\Delta f = 0$.

We introduce the definition of the global exponential state estimator of the uncertain systems (1) as follows.

Definition 1 [9, 10]. For the uncertain systems (1), if there is a dynamic equation

$$E\dot{z}(t) = f(z(t), y(t)), \tag{2}$$

and positive numbers k and α, satisfying

$$\|w(t)\| := \left\| \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \right\| := \|x(t) - z(t)\| \le k \exp(-\alpha t), \quad \forall t \ge 0, \tag{3}$$

the dynamic equation of (2) is called the global exponential state estimator of the uncertain systems (1). At the same time, the positive number α is called the exponential convergence rate.

A lemma is introduced below to facilitate the derivation of subsequent main theorem.

Lemma 1. The global exponential state estimator of the uncertain systems of (1) is as follows:

$$\dot{z}_1(t) = \frac{-\lambda_1}{\lambda_2} z_1(t) + \frac{1}{\lambda_2} y(t), \tag{4a}$$

$$z_2(t) = \frac{-\lambda_1}{\lambda_2} z_1(t) + \frac{1}{\lambda_2} y(t), \quad \forall t \geq 0. \tag{4b}$$

In this case, the guaranteed exponential convergence rate can be estimated as $\alpha := \frac{\lambda_1}{\lambda_2}$.

Proof. From (1), (3), and (4), it can be see that

$$\begin{aligned}
\dot{w}_1(t) &= \dot{x}_1(t) - \dot{z}_1(t) \\
&= x_2(t) - \left[\frac{-\lambda_1}{\lambda_2} z_1(t) + \frac{1}{\lambda_2} y(t)\right] \\
&= \left[\frac{-\lambda_1}{\lambda_2} x_1(t) + \frac{1}{\lambda_2} y(t)\right] - \left[\frac{-\lambda_1}{\lambda_2} z_1(t) + \frac{1}{\lambda_2} y(t)\right] \\
&= \frac{-\lambda_1}{\lambda_2}[x_1(t) - z_1(t)] \\
&= \frac{-\lambda_1}{\lambda_2} w_1(t), \quad \forall t \geq 0.
\end{aligned}$$

It results that

$$w_1(t) = w_1(0) \cdot \exp\left(\frac{-\lambda_1}{\lambda_2} t\right), \quad \forall t \geq 0. \tag{5}$$

In addition, from (1) and (3)-(5), one has

$$\begin{aligned}
w_2(t) &= x_2(t) - z_2(t) \\
&= \left[\frac{-\lambda_1}{\lambda_2} x_1(t) + \frac{1}{\lambda_2} y(t)\right] - \left[\frac{-\lambda_1}{\lambda_2} z_1(t) + \frac{1}{\lambda_2} y(t)\right] \\
&= \frac{-\lambda_1}{\lambda_2}[x_1(t) - z_1(t)] \\
&= \frac{-\lambda_1}{\lambda_2} \cdot w_1(t) \\
&= \frac{-\lambda_1}{\lambda_2} \cdot w_1(0) \cdot \exp\left(\frac{-\lambda_1}{\lambda_2} t\right), \quad \forall t \geq 0.
\end{aligned}$$

As a result, we conclude that

$$\|w(t)\| = \sqrt{w_1^2(t) + w_2^2(t)} = \sqrt{1 + \left(\frac{\lambda_1}{\lambda_2}\right)^2} \cdot |w_1(0)| \cdot \exp\left(\frac{-\lambda_1}{\lambda_2} t\right), \quad \forall t \geq 0. \tag{6}$$

The proof is thus completed. □

CHAOTIC SECURE COMMUNICATION WITH NUMERICAL SIMULATIONS

In this section, we will propose a chaotic secure communication system based on the uncertain jerk systems of (1). Its dynamic equation is shown below, and the block diagram is presented in Fig. 1.

Transmitter:

$$\dot{x}_1(t) = x_2(t), \quad (7a)$$

$$\dot{x}_2(t) = \Delta q_1 x_2(t) + \Delta q_2 x_1^3(t) + \Delta q_3 \cos(wt), + \Delta f(x_1(t), x_2(t)), \quad (7b)$$

$$y(t) = \lambda_1 x_1(t) + \lambda_2 x_2(t), \quad (7c)$$

$$\phi(t) = Lx(t) + m_1(t) + c(t), \ \forall t \geq 0. \quad (7d)$$

Receiver:

$$\dot{z}_1(t) = \frac{-\lambda_1}{\lambda_2} z_1(t) + \frac{1}{\lambda_2} y(t), \quad (8a)$$

$$z_2(t) = \frac{-\lambda_1}{\lambda_2} z_1(t) + \frac{1}{\lambda_2} y(t), \quad (8b)$$

$$m_2(t) = \phi(t) - Lz(t) - c(t), \ \forall t \geq 0, \quad (8c)$$

where $x(t) := [x_1(t) \quad x_2(t)]^T \in \Re^{2\times 1}$, $z(t) := [z_1(t) \quad z_2(t)]^T \in \Re^{2\times 1}$, $m_1(t)$ is the signal to be delivered, $m_2(t)$ is the signal recovered from $m_1(t)$, $L \in \Re^{1\times 2}$ and $c(t)$ are the secret agreement matrix and signal of both parties, respectively. $\Delta q_1, \Delta q_2$, and Δq_3 are uncertain parameters of state equation, $\Delta f(x_1(t), x_2(t))$ are unknown external excitations, and λ_1 and λ_2 are parameters of output equation with $\lambda_1 \cdot \lambda_2 > 0$. As we know, a high-quality chaotic secure communication system must satisfy the requirement that the signal $m_1(t)$ transmitted by the transmitter and the signal $m_2(t)$ restored by the receiver are exactly the same or similar. In the following, for the sake of convenience, we define the error signal as $e(t) := m_2(t) - m_1(t)$.

The main theorem of this paper is introduced as follows.

Theorem 1. The error signal $e(t)$ of the secure communication system (7) with (8) satisfies

$$|e(t)| \leq \|L\| \cdot \sqrt{1 + \left(\frac{\lambda_1}{\lambda_2}\right)^2} \cdot |w_1(0)| \cdot \exp\left(\frac{-\lambda_1}{\lambda_2} t\right), \quad \forall t \geq 0.$$

More specifically, the error signal will approach zero in the form of exponential convergence rate $\frac{\lambda_1}{\lambda_2}$.

Proof. By Lemma 1 with (3) and (6)-(8), one can see that

$$\begin{aligned}
|e(t)| &= |m_2(t) - m_1(t)| \\
&= \left| [\phi(t) - Lz(t) - c(t)] \; - [\phi(t) - Lx(t) - c(t)] \right| \\
&= \left| L[x(t) - z(t)] \right| \\
&\le \|L\| \cdot \|w(t)\| \\
&\le \|L\| \cdot \sqrt{1 + \left(\frac{\lambda_1}{\lambda_2}\right)^2} \cdot |w_1(0)| \cdot \exp\left(\frac{-\lambda_1}{\lambda_2} t\right), \quad \forall\, t \ge 0.
\end{aligned}$$

This means that we can recover the message $m_1(t)$ at the receiving end. The proof is thus completed. □

Example: Consider chaotic secure communication systems (7) and (8) with $\lambda_1 = 3, \lambda_2 = 1, L = [1 \;\; 1], c(t) = 2\sin(3t)$. When the signal $m_1(t)$ is transmitted as shown in Figure 2, the recoved signal $m_2(t)$ obtained by the receiver is as shown in Figure 3. The error signal $m_2(t) - m_1(t)$ between the two signals is shown in Figure 4. From Figure 4, we know that synchronization of $m_1(t)$ and $m_2(t)$ can be achieved in about two seconds. Besides, by Theorem 1, we know that this error signal approaches zero with an exponential convergence rate of 3.

Remark 2. The communication system mentioned in Theorem 1 has the multiple security effects of chaos security, parameter and signal protocols at the same time.

CONCLUSION

In this paper, a global exponential state estimator for uncertain jerk chaotic systems has been proposed. Based on the above state estimators, a chaotic secure communication system with multiple security effects has been planned to be designed. Both the transmission signal and recovery signal of such a secure communication system have been verified to be synchronized in exponential convergence mode, and the exponential convergence rate has also been derived. Finally, multiple numerical simulation results have been supplemented to verify and illustrate the correctness of the main theorem.

ACKNOWLEDGEMENT

The authors thank the Ministry of Science and Technology of Republic of China for supporting this work under grant MOST 109-2221-E-214-014.

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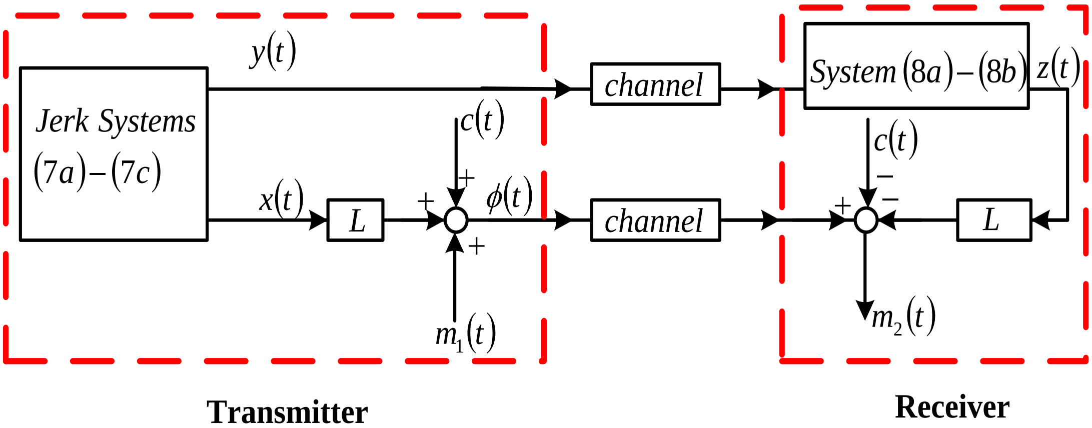

Figure 1: Block diagram of the chaotic secure communication system.

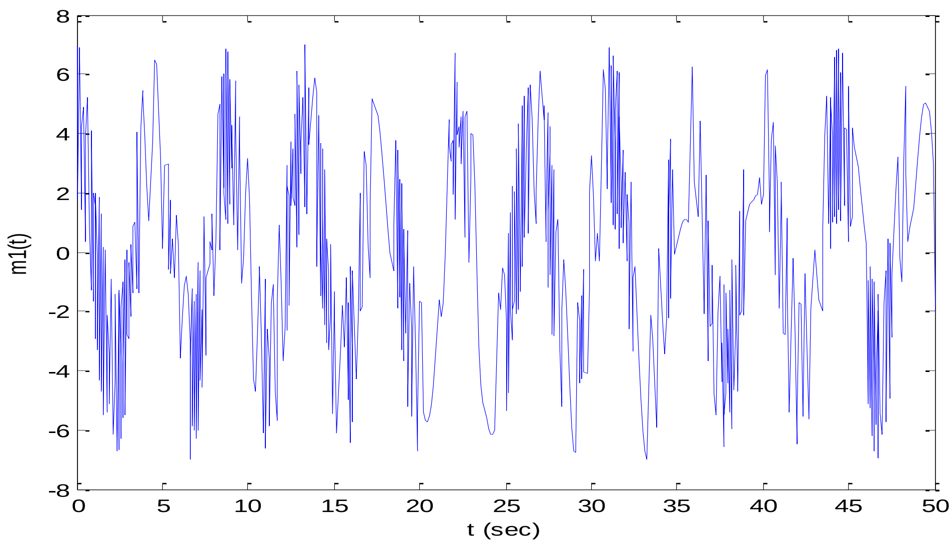

Figure 2: The signal $m_1(t)$ transmitted by the transmitter.

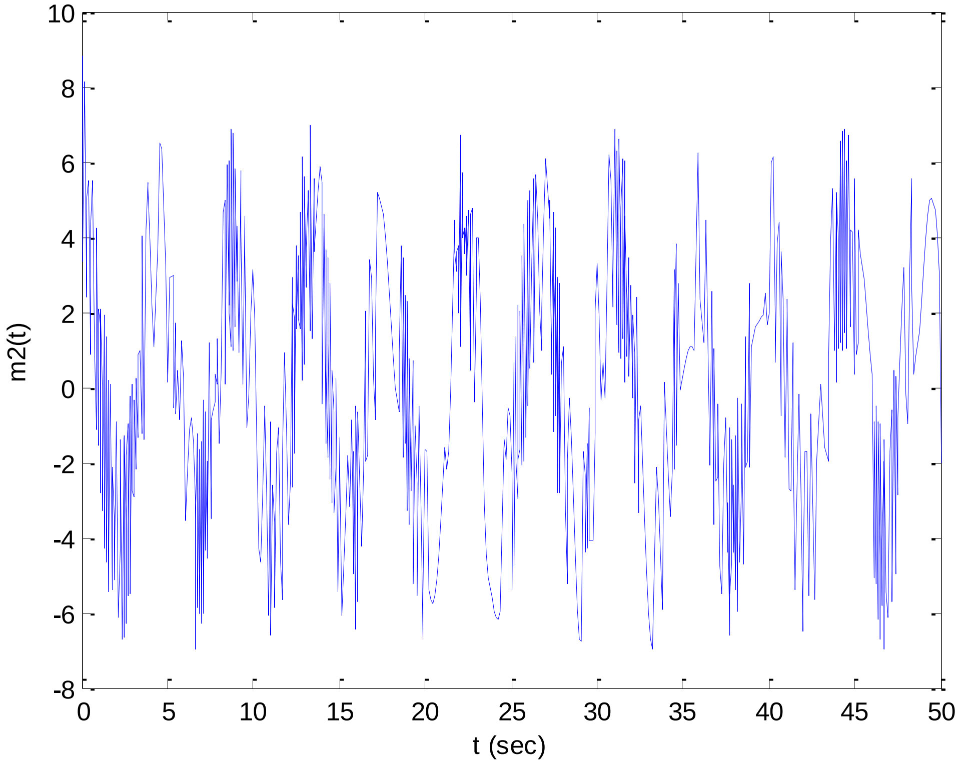

Figure 3: Recoverd signal of $m_2(t)$ described in the receiver.

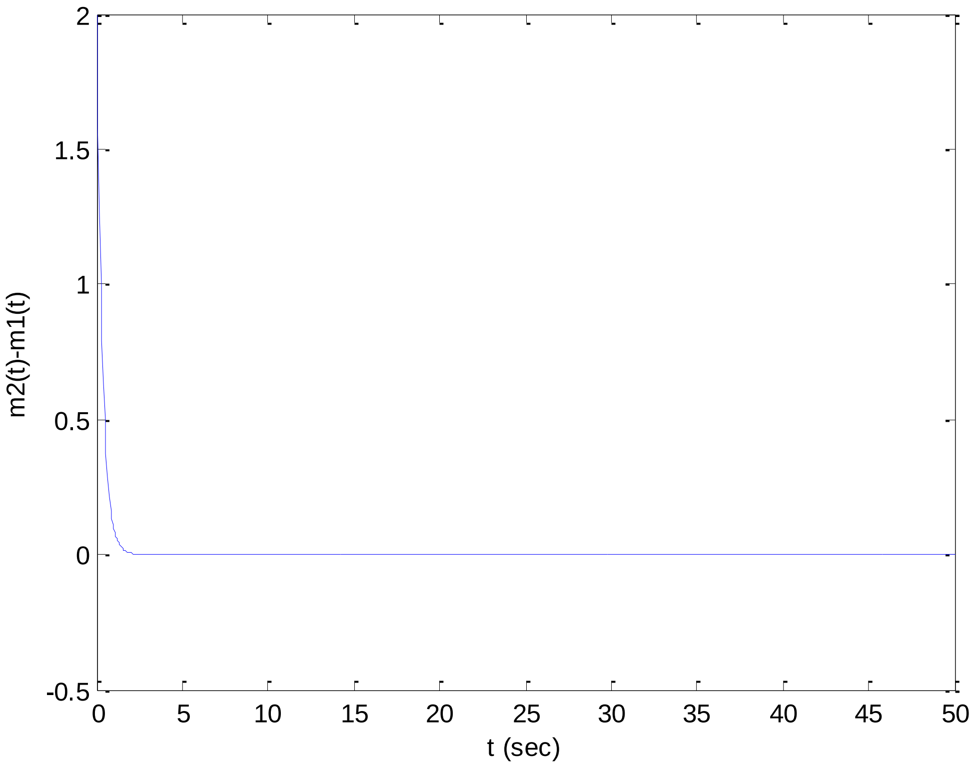

Figure 4: Error signal of $m_2(t) - m_1(t)$.