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ROBUST CONTROL DESIGN FOR A CLASS OF UNCERTAIN FIFTH-ORDER LASER SYSTEMS

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ABSTRACT

This paper aims at a class of uncertain 5D laser dynamic systems, seeking a linear controller that is cheap and easy to produce in hardware, so that the entire system can achieve stability. Using differential and integral inequalities combined with control theory, a linear controller guaranteed to achieve global exponential stability will be derived. Besides, the guaranteed exponential convergence rate will be estimated. Finally, multiple numerical simulation results are provided to illustrate the correctness of the main theorem of this article and the design process of the linear controller.

Keywords: uncertain 5D laser system, robust control, linear control, state feedback, global exponential stability, uncertain systems.

INTRODUCTION

As we know, since laser dynamic systems are nonlinear systems, their related analysis and design are naturally more complex and difficult than linear systems. In recent years, there have been many related analyzes and research results on laser dynamic systems; see, for example, [1]-[11] and the references therein. In particular, some laser dynamic systems have been proven to be chaotic systems, and the unpredictability of their signals makes controller design more difficult.

The parameters of most dynamic systems often change with temperature, humidity or pressure, etc. Therefore, it seems that a mathematical model that is consistent with the actual situation becomes crucial. Considering the uncertain dynamic system as a real model can be the solution.

In this paper, due to the above motivations, the problem of controller design for uncertain laser dynamic systems is explored. For a type of uncertain 5D laser system, we hope to design a linear controller to promote the entire closed-loop system to achieve global exponential stability. In addition, we will accurately calculate its exponential convergence rate. Finally, some numerical simulation results are provided to verify the correctness of the proposed theory. Throughout this paper, $|a|$ denotes the modulus of a complex number a and $\|x\|$ means the Euclidean norm of the vector $x \in \Re^n$.

PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we explore the following uncertain 5D laser systems:

$$\dot{x}_1 = \Delta a x_1 + \Delta d_1 x_2 + \Delta d_2 x_3 + f_1(x_1, x_2, x_3, x_4, x_5), \tag{1a}$$

$$\dot{x}_2 = \Delta d_3 x_1 + \Delta d_4 x_2 + \Delta d_5 x_3 + \Delta d_6 x_4 + \Delta d_7 x_5 + f_2(x_1, x_2, x_3, x_4, x_5) + \Delta\phi_1(u_1), \tag{1b}$$

$$\dot{x}_3 = \Delta d_8 x_1 + \Delta d_9 x_2 + \Delta d_{10} x_3 + \Delta d_{11} x_4 + \Delta d_{12} x_5 + f_3(x_1, x_2, x_3, x_4, x_5) + \Delta\phi_2(u_2), \tag{1c}$$

$$\dot{x}_4 = \Delta d_{13} x_2 + \Delta d_{14} x_3 + \Delta b x_4 + f_4(x_1, x_2, x_3, x_4, x_5), \tag{1d}$$

$$\dot{x}_5 = \Delta d_{15} x_2 + \Delta d_{16} x_3 + \Delta c x_5 + f_5(x_1, x_2, x_3, x_4, x_5), \tag{1e}$$

where $x(t) := [x_1(t) \;\; x_2(t) \;\; x_3(t) \;\; x_4(t) \;\; x_5(t)]^T \in \Re^{5\times 1}$ is the state vector, $u(t) := [u_1(t) \;\; u_2(t)]^T \in \Re^{2\times 1}$ is the input vector, $\Delta a, \Delta b, \Delta c,$ and Δd_i are uncertain parameters, f_i is a nonlinear smooth function and satisfies $\Delta f_i(0,0,0,0,0) = 0, \forall\, i \in \{1,2,3,4,5\}$, and the smooth operator $\Delta\phi_i(u): \Re \to \Re, \forall\, i \in \{1,2\}$ is the uncertain input nonlinearity.

Below we make appropriate assumptions regarding the nonlinear terms and uncertain terms of uncertain nonlinear systems of (1):

(A1) There are constants $\underline{a}, \underline{b}, \underline{c},$ and $\overline{d_i}$ such that

$$\Delta a \le -\underline{a} < 0, \quad \Delta b \le -\underline{b} < 0, \quad \Delta c \le -\underline{c} < 0, \quad |\Delta d_i| \le \overline{d_i}, \quad \forall\, i \in \{1,2,3,\cdots,16\}.$$

(A2) There are positive numbers $k_1, k_2, k_3, k_4,$ and k_5 such that

$$\sum_{i=1}^{5} k_i^2 \cdot x_i \cdot f_i(x_1, x_2, x_3, x_4, x_5) = 0.$$

(A3) There are positive numbers r_1 and r_2 such that

$$r_i \cdot u^2 \le u \cdot \Delta\phi_i(u), \quad \forall\, i \in \{1, 2\}.$$

Remark 1: The 5D laser dynamic system was first proposed and studied by Zeghlach and Mandel [11]. Its model is equivalent to (1) with the following parameters:

$$\Delta a = -2,\ \Delta b = -1,\ \Delta c = -0.25,\ \ \Delta d_1 = -\Delta d_3 = 2\Delta d_{14} = -2\Delta d_{11} = -0.004,$$

$$\Delta d_4 = -\Delta d_2 = -\Delta d_6 = 2\Delta d_{10} = -2,\ \ \Delta d_8 = \Delta d_{13} = 20,$$

$$\Delta d_i = 0,\ \forall\, i \in \{5,7,9,12,15,16\},\ \Delta\phi_i(u) = 0,\ \forall\, i \in \{1,2\}.$$

Obviously, the system considered in [11] can be regarded as a special case of uncertain systems of (1).

The following introduces the relevant definitions mentioned in the main theorem of this paper.

Definition 1 [12, 13]. If there are positive numbers α, k and a suitable controller satisfies

$$\|x(t)\| \le k \cdot e^{-\alpha t}, \quad \forall t \ge 0,$$

the uncertain nonlinear systems (1) are said to be globally exponentially stabilized. Furthermore, we call this positive number α the exponential convergence rate.

Before stating the main theorem, we define the following two parameters:

$$l_1 := \frac{3\left(\frac{k_1\overline{d_1}}{k_2}+\frac{k_2\overline{d_3}}{k_1}\right)^2}{4\underline{a}}+\frac{3\left(\frac{k_2\overline{d_6}}{k_4}+\frac{k_4\overline{d_{13}}}{k_2}\right)^2}{4\underline{b}}+\frac{3\left(\frac{k_2\overline{d_7}}{k_5}+\frac{k_5\overline{d_{15}}}{k_2}\right)^2}{4\underline{c}}+\frac{\left(\overline{d_5}+\frac{k_3^2\overline{d_9}}{k_2^2}\right)}{2}, \tag{2a}$$

$$l_2 := \frac{3\left(\frac{k_1\overline{d_2}}{k_3}+\frac{k_3\overline{d_8}}{k_1}\right)^2}{4\underline{a}}+\frac{3\left(\frac{k_3\overline{d_{11}}}{k_4}+\frac{k_4\overline{d_{14}}}{k_3}\right)^2}{4\underline{b}}+\frac{3\left(\frac{k_3\overline{d_{12}}}{k_5}+\frac{k_5\overline{d_{16}}}{k_3}\right)^2}{4\underline{c}}+\frac{\left(\overline{d_9}+\frac{k_2^2\overline{d_5}}{k_3^2}\right)}{2}. \tag{2b}$$

In the following we present the main result for the global exponential stabilization of uncertain nonlinear systems of (1).

Theorem 1. The uncertain 5D laser systems (1) with (A1)-(A3) are globally exponentially stable at the zero equilibrium point, subject to the linear controller

$$u = \left[-\beta_1 x_2 \quad -\beta_2 x_3\right]^T, \tag{3}$$

with

$$\beta_1 \ge \frac{l_1+\overline{d_4}+h_1}{r_1}, \quad \beta_2 \ge \frac{l_2+\overline{d_{10}}+h_2}{r_2}, \tag{4}$$

$h_1 > 0$, and $h_2 > 0$. Besides, the guaranteed exponential convergence rate can be estimated as

$$\alpha := \min\left\{\frac{\underline{a}}{3}, \frac{\underline{b}}{3}, \frac{\underline{c}}{3}, h_1, h_2\right\}. \tag{5}$$

Proof. Let

$$V(x(t)) := \sum_{i=1}^{5} k_i^2 \cdot x_i^2(t). \tag{6}$$

The derivative of $V(x(t))$ with respect to time along the trajectories of uncertain systems (1), with (A1)-(A3) and (2)-(6), can be derived as

$$\begin{aligned}
\dot V(x(t)) &= 2k_1^2 \cdot x_1\dot x_1 + 2k_2^2 \cdot x_2\dot x_2 + 2k_3^2 \cdot x_3\dot x_3 + 2k_4^2 \cdot x_4\dot x_4 + 2k_5^2 \cdot x_5\dot x_5 \\
&= 2k_1^2x_1\left(\Delta a x_1 + \Delta d_1 x_2 + \Delta d_2 x_3 + f_1\right) \\
&\quad + 2k_2^2x_2\left[\Delta d_3 x_1 + \Delta d_4 x_2 + \Delta d_5 x_3 + \Delta d_6 x_4 + \Delta d_7 x_5 + f_2 + \Delta\phi_1(u_1)\right] \\
&\quad + 2k_3^2x_3\left[\Delta d_8 x_1 + \Delta d_9 x_2 + \Delta d_{10} x_3 + \Delta d_{11} x_4 + \Delta d_{12} x_5 + f_3 + \Delta\phi_2(u_2)\right] \\
&\quad + 2k_4^2x_4\left(\Delta d_{13} x_2 + \Delta d_{14} x_3 + \Delta b x_4 + f_4\right) \\
&\quad + 2k_5^2x_5\left(\Delta d_{15} x_2 + \Delta d_{16} x_3 + \Delta c x_5 + f_5\right) \\
&\le -2k_1^2\underline{a}x_1^2 + 2k_1^2\overline{d_1}|x_1||x_2| + 2k_1^2\overline{d_2}|x_1||x_3| \\
&\quad + 2k_2^2\overline{d_3}|x_1||x_2| + 2k_2^2\overline{d_4}x_2^2 + 2k_2^2\overline{d_5}|x_2||x_3| \\
&\quad + 2k_2^2\overline{d_6}|x_2||x_4| + 2k_2^2\overline{d_7}|x_2||x_5| + 2k_3^2\overline{d_8}|x_1||x_3|
\end{aligned}$$

$$
\begin{aligned}
&+2k_3^2\overline{d_9}|x_2||x_3|+2k_3^2\overline{d_{10}}x_3^2+2k_3^2\overline{d_{11}}|x_3||x_4|+2k_3^2\overline{d_{12}}|x_3||x_5| \\
&+2k_4^2\overline{d_{13}}|x_2||x_4|+2k_4^2\overline{d_{14}}|x_3||x_4|-2k_4^2\underline{b}x_4^2 \\
&+2k_5^2\overline{d_{15}}|x_2||x_5|+2k_5^2\overline{d_{16}}|x_3||x_5|-2k_5^2\underline{c}x_5^2 \\
&+2\left(k_1^2x_1f_1+k_2^2x_2f_2+k_3^2x_3f_3+k_4^2x_4f_4+k_5^2x_5f_5\right) \\
&+2k_2^2x_2\Delta\phi_1(u_1)+2k_3^2x_3\Delta\phi_2(u_2)
\end{aligned}
$$

$$
\begin{aligned}
&=-2k_1^2\left(\frac{\underline{a}}{3}+\frac{\underline{a}}{3}+\frac{\underline{a}}{3}\right)x_1^2+2k_1^2\overline{d_1}|x_1||x_2|+2k_1^2\overline{d_2}|x_1||x_3| \\
&+2k_2^2\overline{d_3}|x_1||x_2|+2k_2^2\overline{d_4}x_2^2+2k_2^2\overline{d_5}|x_2||x_3| \\
&+2k_2^2\overline{d_6}|x_2||x_4|+2k_2^2\overline{d_7}|x_2||x_5|+2k_3^2\overline{d_8}|x_1||x_3| \\
&+2k_3^2\overline{d_9}|x_2||x_3|+2k_3^2\overline{d_{10}}x_3^2+2k_3^2\overline{d_{11}}|x_3||x_4|+2k_3^2\overline{d_{12}}|x_3||x_5| \\
&+2k_4^2\overline{d_{13}}|x_2||x_4|+2k_4^2\overline{d_{14}}|x_3||x_4|-2k_4^2\left(\frac{\underline{b}}{3}+\frac{\underline{b}}{3}+\frac{\underline{b}}{3}\right)x_4^2 \\
&+2k_5^2\overline{d_{15}}|x_2||x_5|+2k_5^2\overline{d_{16}}|x_3||x_5|-2k_5^2\left(\frac{\underline{c}}{3}+\frac{\underline{c}}{3}+\frac{\underline{c}}{3}\right)x_5^2 \\
&+2k_2^2x_2\Delta\phi_1(u_1)+2k_3^2x_3\Delta\phi_2(u_2)
\end{aligned}
$$

$$
\begin{aligned}
&=-2k_1^2\left(\frac{\underline{a}}{3}+\frac{\underline{a}}{3}+\frac{\underline{a}}{3}\right)x_1^2+2k_1^2\overline{d_1}|x_1||x_2|+2k_1^2\overline{d_2}|x_1||x_3| \\
&+2k_2^2\overline{d_3}|x_1||x_2|+2k_2^2\overline{d_4}x_2^2+2k_2^2\overline{d_5}|x_2||x_3| \\
&+2k_2^2\overline{d_6}|x_2||x_4|+2k_2^2\overline{d_7}|x_2||x_5|+2k_3^2\overline{d_8}|x_1||x_3| \\
&+2k_3^2\overline{d_9}|x_2||x_3|+2k_3^2\overline{d_{10}}x_3^2+2k_3^2\overline{d_{11}}|x_3||x_4|+2k_3^2\overline{d_{12}}|x_3||x_5| \\
&+2k_4^2\overline{d_{13}}|x_2||x_4|+2k_4^2\overline{d_{14}}|x_3||x_4|-2k_4^2\left(\frac{\underline{b}}{3}+\frac{\underline{b}}{3}+\frac{\underline{b}}{3}\right)x_4^2 \\
&+2k_5^2\overline{d_{15}}|x_2||x_5|+2k_5^2\overline{d_{16}}|x_3||x_5|-2k_5^2\left(\frac{\underline{c}}{3}+\frac{\underline{c}}{3}+\frac{\underline{c}}{3}\right)x_5^2 \\
&-2k_2^2\frac{u_1\Delta\phi_1(u_1)}{\beta_1}-2k_3^2\frac{u_2\Delta\phi_2(u_2)}{\beta_2}
\end{aligned}
$$

$$
\begin{aligned}
&\le-2k_1^2\left(\frac{\underline{a}}{3}+\frac{\underline{a}}{3}+\frac{\underline{a}}{3}\right)x_1^2+2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2|+2\left(k_1^2\overline{d_2}+k_3^2\overline{d_8}\right)|x_1||x_3| \\
&+2k_2^2\overline{d_4}x_2^2+2\left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)|x_2||x_3|+2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{13}}\right)|x_2||x_4| \\
&+2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{15}}\right)|x_2||x_5|+2k_3^2\overline{d_{10}}x_3^2+2\left(k_3^2\overline{d_{11}}+k_4^2\overline{d_{14}}\right)|x_3||x_4| \\
&+2\left(k_3^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_3||x_5|-2k_4^2\left(\frac{\underline{b}}{3}+\frac{\underline{b}}{3}+\frac{\underline{b}}{3}\right)x_4^2-2k_5^2\left(\frac{\underline{c}}{3}+\frac{\underline{c}}{3}+\frac{\underline{c}}{3}\right)x_5^2 \\
&-2k_2^2\frac{r_1u_1^2}{\beta_1}-2k_3^2\frac{r_2u_2^2}{\beta_2}
\end{aligned}
$$

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$$
\begin{aligned}
&= -2k_1^2\left(\frac{a}{3}+\frac{a}{3}+\frac{a}{3}\right)x_1^2 + 2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2}+k_3^2\overline{d_8}\right)|x_1||x_3| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{13}}\right)|x_2||x_4| \\
&\quad + 2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{15}}\right)|x_2||x_5| + 2k_3^2\overline{d_{10}}x_3^2 + 2\left(k_3^2\overline{d_{11}}+k_4^2\overline{d_{14}}\right)|x_3||x_4| \\
&\quad + 2\left(k_3^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_3||x_5| - 2k_4^2\left(\frac{b}{3}+\frac{b}{3}+\frac{b}{3}\right)x_4^2 - 2k_5^2\left(\frac{c}{3}+\frac{c}{3}+\frac{c}{3}\right)x_5^2 \\
&\quad - 2k_2^2 r_1\beta_1 x_2^2 - 2k_3^2 r_2\beta_2 x_3^2
\end{aligned}
$$

$$
\begin{aligned}
&\le -2k_1^2\left(\frac{a}{3}+\frac{a}{3}+\frac{a}{3}\right)x_1^2 + 2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2| + 2\left(k_1^2\overline{d_2}+k_3^2\overline{d_8}\right)|x_1||x_3| \\
&\quad + 2k_2^2\overline{d_4}x_2^2 + 2\left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)|x_2||x_3| + 2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{13}}\right)|x_2||x_4| \\
&\quad + 2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{15}}\right)|x_2||x_5| + 2k_3^2\overline{d_{10}}x_3^2 + 2\left(k_3^2\overline{d_{11}}+k_4^2\overline{d_{14}}\right)|x_3||x_4| \\
&\quad + 2\left(k_3^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_3||x_5| - 2k_4^2\left(\frac{b}{3}+\frac{b}{3}+\frac{b}{3}\right)x_4^2 - 2k_5^2\left(\frac{c}{3}+\frac{c}{3}+\frac{c}{3}\right)x_5^2 \\
&\quad - \frac{3\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)^2}{2\underline{a}k_1^2}x_2^2 - \frac{3\left(k_2^2\overline{d_6}+k_4^2\overline{d_{13}}\right)^2}{2\underline{b}k_4^2}x_2^2 - \frac{3\left(k_2^2\overline{d_7}+k_5^2\overline{d_{15}}\right)^2}{2\underline{c}k_5^2}x_2^2 \\
&\quad - \left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)x_2^2 - 2\overline{d_4}k_2^2x_2^2 - 2h_1k_2^2x_2^2 \\
&\quad - \frac{3\left(k_1^2\overline{d_2}+k_3^2\overline{d_8}\right)^2}{2\underline{a}k_1^2}x_3^2 - \frac{3\left(k_3^2\overline{d_{11}}+k_4^2\overline{d_{14}}\right)^2}{2\underline{b}k_4^2}x_3^2 - \frac{3\left(k_3^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)^2}{2\underline{c}k_5^2}x_3^2 \\
&\quad - \left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)x_3^2 - 2\overline{d_{10}}k_3^2x_3^2 - 2h_2k_3^2x_3^2
\end{aligned}
$$

$$
\begin{aligned}
&= -2k_1^2\left(\frac{a}{3}\right)x_1^2 + 2\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_1||x_2| - \frac{3\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)^2}{2\underline{a}k_1^2}x_2^2 \\
&\quad - 2k_1^2\left(\frac{a}{3}\right)x_1^2 + 2\left(k_1^2\overline{d_2}+k_3^2\overline{d_8}\right)|x_1||x_3| - \frac{3\left(k_1^2\overline{d_2}+k_3^2\overline{d_8}\right)^2}{2\underline{a}k_1^2}x_3^2 \\
&\quad - \left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)x_2^2 + 2\left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)|x_2||x_3| - \left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)x_3^2 \\
&\quad - \frac{3\left(k_2^2\overline{d_6}+k_4^2\overline{d_{13}}\right)^2}{2\underline{b}k_4^2}x_2^2 + 2\left(k_2^2\overline{d_6}+k_4^2\overline{d_{13}}\right)|x_2||x_4| - 2k_4^2\left(\frac{b}{3}\right)x_4^2 \\
&\quad - \frac{3\left(k_2^2\overline{d_7}+k_5^2\overline{d_{15}}\right)^2}{2\underline{c}k_5^2}x_2^2 + 2\left(k_2^2\overline{d_7}+k_5^2\overline{d_{15}}\right)|x_2||x_5| - 2k_5^2\left(\frac{c}{3}\right)x_5^2 \\
&\quad - \frac{3\left(k_3^2\overline{d_{11}}+k_4^2\overline{d_{14}}\right)^2}{2\underline{b}k_4^2}x_3^2 + 2\left(k_3^2\overline{d_{11}}+k_4^2\overline{d_{14}}\right)|x_3||x_4| - 2k_4^2\left(\frac{b}{3}\right)x_4^2 \\
&\quad - \frac{3\left(k_3^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)^2}{2\underline{c}k_5^2}x_3^2 + 2\left(k_3^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_3||x_5| - 2k_5^2\left(\frac{c}{3}\right)x_5^2 \\
&\quad - 2k_1^2\left(\frac{a}{3}\right)x_1^2 - 2k_2^2h_1x_2^2 - 2k_3^2h_2x_3^2 - 2k_4^2\left(\frac{b}{3}\right)x_4^2 - 2k_5^2\left(\frac{c}{3}\right)x_5^2
\end{aligned}
$$

$$
\begin{aligned}
&= -\left[\sqrt{2\left(\frac{\underline{a}}{3}\right)}\cdot k_1|x_1| - \sqrt{\frac{3}{2\underline{a}}}\cdot\frac{\left(k_1^2\overline{d_1}+k_2^2\overline{d_3}\right)|x_2|}{k_1}\right]^2 \\
&\quad -\left[\sqrt{2\left(\frac{\underline{a}}{3}\right)}\cdot k_1|x_1| - \sqrt{\frac{3}{2\underline{a}}}\cdot\frac{\left(k_1^2\overline{d_2}+k_3^2\overline{d_8}\right)|x_3|}{k_1}\right]^2 \\
&\quad -\left[\sqrt{\left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)}\cdot|x_2| - \sqrt{\left(k_2^2\overline{d_5}+k_3^2\overline{d_9}\right)}\cdot|x_3|\right]^2 \\
&\quad -\left[\sqrt{2\left(\frac{\underline{b}}{3}\right)}\cdot k_4|x_4| - \sqrt{\frac{3}{2\underline{b}}}\cdot\frac{\left(k_2^2\overline{d_6}+k_4^2\overline{d_{13}}\right)|x_2|}{k_4}\right]^2 \\
&\quad -\left[\sqrt{2\left(\frac{\underline{c}}{3}\right)}\cdot k_5|x_5| - \sqrt{\frac{3}{2\underline{c}}}\cdot\frac{\left(k_2^2\overline{d_7}+k_5^2\overline{d_{15}}\right)|x_2|}{k_5}\right]^2 \\
&\quad -\left[\sqrt{2\left(\frac{\underline{b}}{3}\right)}\cdot k_4|x_4| - \sqrt{\frac{3}{2\underline{b}}}\cdot\frac{\left(k_3^2\overline{d_{11}}+k_4^2\overline{d_{14}}\right)|x_3|}{k_4}\right]^2 \\
&\quad -\left[\sqrt{2\left(\frac{\underline{c}}{3}\right)}\cdot k_5|x_5| - \sqrt{\frac{3}{2\underline{c}}}\cdot\frac{\left(k_3^2\overline{d_{12}}+k_5^2\overline{d_{16}}\right)|x_3|}{k_5}\right]^2 \\
&\quad -2k_1^2\left(\frac{\underline{a}}{3}\right)x_1^2 - 2k_2^2h_1x_2^2 - 2k_3^2h_2x_3^2 - 2k_4^2\left(\frac{\underline{b}}{3}\right)x_4^2 - 2k_5^2\left(\frac{\underline{c}}{3}\right)x_5^2 \\
&\le -2k_1^2\left(\frac{\underline{a}}{3}\right)x_1^2 - 2k_2^2h_1x_2^2 - 2k_3^2h_2x_3^2 - 2k_4^2\left(\frac{\underline{b}}{3}\right)x_4^2 - 2k_5^2\left(\frac{\underline{c}}{3}\right)x_5^2 \\
&\le -2\alpha\left(k_1^2x_1^2+k_2^2x_2^2+k_3^2x_3^2+k_4^2x_4^2+k_5^2x_5^2\right) \\
&= -2\alpha V, \quad \forall\, t\ge 0.
\end{aligned}
$$

Thus, one has

$$e^{2\alpha t}\cdot\dot{V}+e^{2\alpha t}\cdot 2\alpha\cdot V=\frac{d}{dt}\left[e^{2\alpha t}\cdot V\right]\le 0,\quad \forall\, t\ge 0.$$

It follows that

$$\int_0^t \frac{d}{d\tau}\left[e^{2\alpha\tau}\cdot V(x(t))\right]d\tau = e^{2\alpha t}\cdot V(x(t))-V(x(0))\le\int_0^t 0\,d\tau=0,\quad \forall\, t\ge 0. \tag{7}$$

From (6) and (7), it can be readily obtained that

$$\left(\min_{1\le i\le 5} k_i\right)^2\|x(t)\|^2\le V(x(t))\le e^{-2\alpha t}V(x(0)),\ \forall\, t\ge 0,$$

In this way we can get

$$\|x(t)\|\le\frac{\sqrt{V(x(0))}}{\left(\min_{1\le i\le 5} k_i\right)}\cdot e^{-\alpha t},\ \forall\, t\ge 0.$$

The proof is thus completed.

Remark 2. Due to the proposed linear controller of (3), it is not only cheap but also easy to implement in hardware.

NUMERICAL SIMULATIONS

In this section, we consider the uncertain 5D laser systems of (1) with

$$f_1 = 0, \quad f_2 = 0, \quad f_3 = -x_1 x_5, \quad f_4 = -x_2 x_5, \quad f_5 = x_1 x_3 + x_2 x_4, \tag{8a}$$

$$\Delta\phi_1(u_1) = \Delta d_{17} u_1 + \Delta d_{18} u_1^7, \quad \Delta\phi_2(u_2) = \Delta d_{19} u_2 + \Delta d_{20} u_2^3 \tag{8b}$$

$$10 \le \Delta d_{17} \le 12, \quad 0 \le \Delta d_{18} \le 2, \quad 5 \le \Delta d_{19} \le 7, \quad 0 \le \Delta d_{20} \le 3, \tag{8c}$$

$$\underline{a} = 1, \quad \underline{b} = 0.5, \quad \underline{c} = 0.2, \quad \overline{d_i} = 1, \quad \forall\, i \in \{1,3,5,7,9,11,12,14,15,16\}, \tag{8d}$$

$$\overline{d_2} = \overline{d_4} = \overline{d_6} = 3, \quad \overline{d_8} = \overline{d_{13}} = 21, \quad \overline{d_{10}} = 2. \tag{8e}$$

By virtue of choosing parameters $r_1 = 10$ and $r_2 = 5$ with (8b) and (8c), (A3) is evidently consistent. Obviously, by selecting parameters $k_1 = k_2 = k_3 = k_4 = k_5 = 1$, (A2) is undoubtedly satisfied. From (2), (8e), and choosing parameters $h_1 = h_2 = 1$, it yields $\frac{l_1 + \overline{d_4} + h_1}{r_1} = 88.7$ and $\frac{l_2 + \overline{d_{10}} + h_2}{r_2} = 91.4$. Therefore, according to Theorem 1 with $\beta_1 = 89$ and $\beta_2 = 92$, we conclude that the uncertain nonlinear systems (1) with (8) subject to the linear control

$$u = [-89 x_2 \quad -92 x_3]^T \tag{9}$$

are globally exponentially stable. At the same time, by virtue of (5), the guaranteed exponential convergence rate can be estimated as

$$\alpha = \min\left\{\frac{\underline{a}}{3}, \frac{\underline{b}}{3}, \frac{\underline{c}}{3}, h_1, h_2\right\} = \frac{1}{15}.$$

State variables trajectories of uncontrolled and feedback-controlled systems are displayed in Figure 1 and 2, respectively. From Figure 2, it can be seen that through the linear controller of (9), the system (1) with (8) can attain the goal of global exponential stability. Meanwhile, the input signal trajectories and the hardware implementation diagram of the proposed linear controller are shown in Figure 3 and Figure 4 respectively.

CONCLUSION

In this paper, the controller design problem for a class of uncertain fifth-order nonlinear control systems has been explored. Combining the theory of differential and integral inequalities, a simple linear controller has been proposed to promote a class of nonlinear control systems with multiple uncertainties to achieve the goal of global exponential stability. In addition, the guaranteed exponential convergence rate of such uncertain nonlinear systems has been precisely calculated. Finally, some numerical simulation results have also been presented to verify and illustrate the correctness of this main theorem and the design process of the controller.

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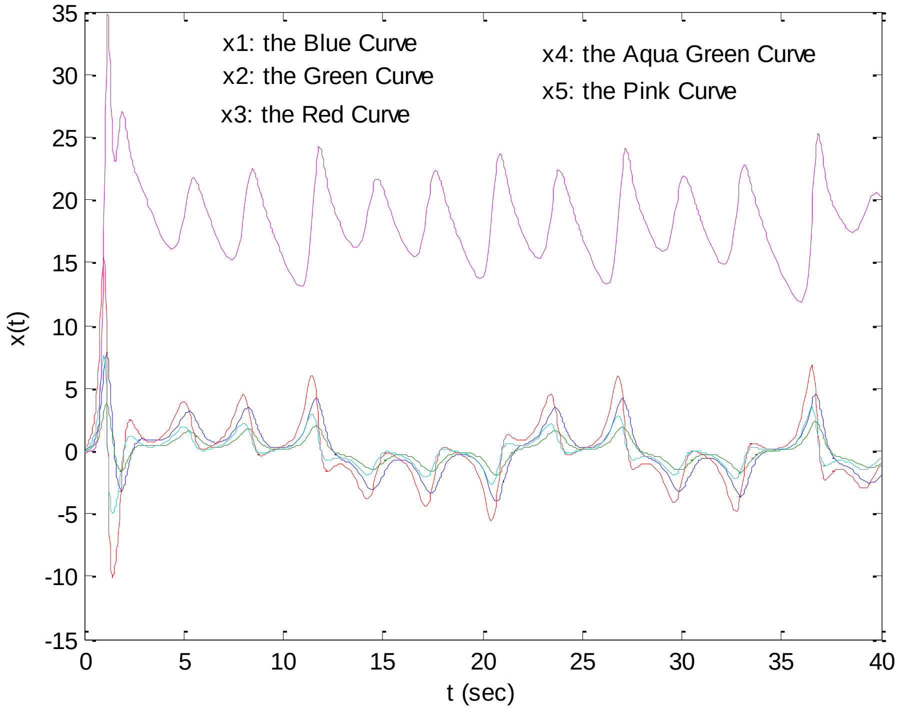

Figure 1: State variables trajectories of the uncontrolled systems of (1) with (8).

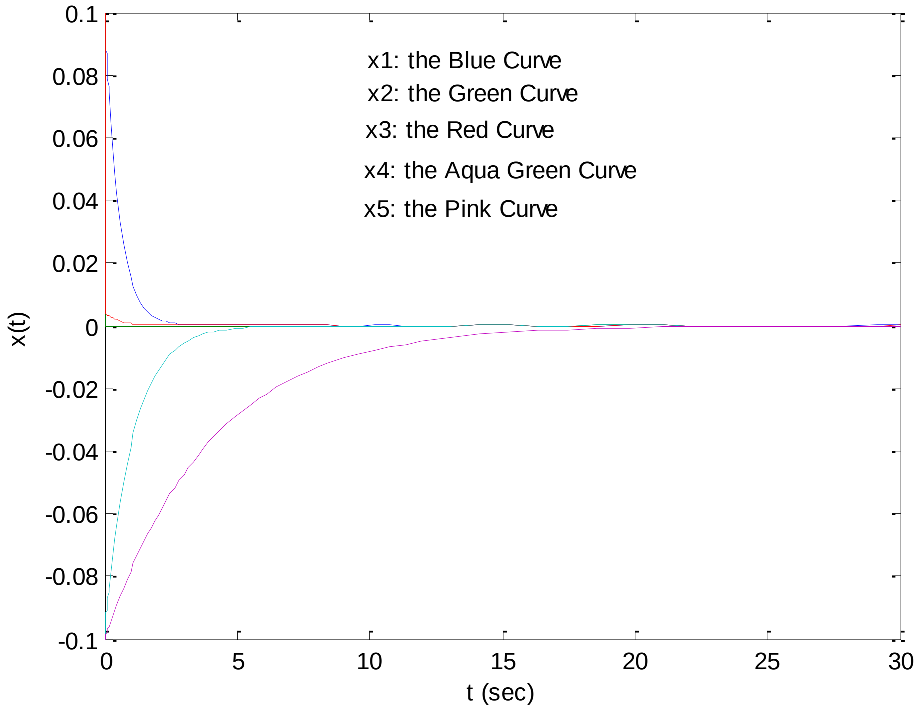

Figure 2: State variables trajectories of the feedback-controlled systems of (1) with (8) and (9).

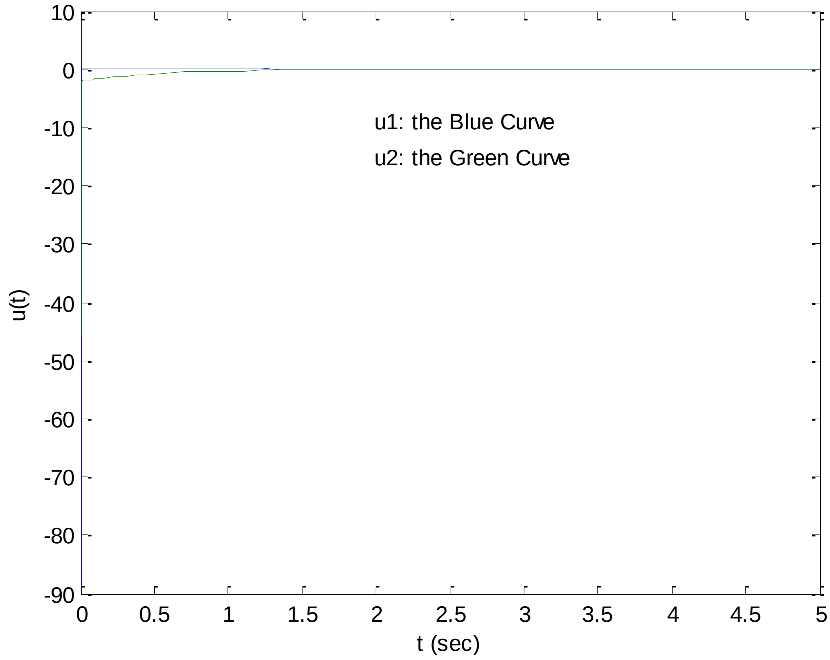

Figure 3: The time response of the control signal of (9).

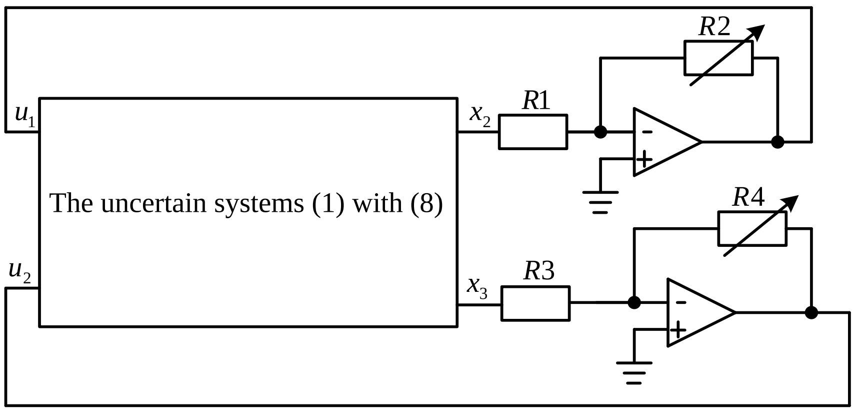

Figure 4: The diagram of implementation of numerical example, where $R1 = 1k\Omega$, $R2 = 89k\Omega$, $R3 = 1k\Omega$, , and $R4 = 92k\Omega$.