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WONDERFUL STRAIGHT LINES AND PLANES OF TRIHEDRONS

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ABSTRACT

In this article, some information about three-sided angles that are not often found. This article explores the wonderful straight lines and planes of Trihedrons in order for the reader to visualize spatial bodies and form a conic about them.

Keywords: Median plane of a trihedron, axis of a circular cone, internal and external cone axes, Angle, inequality, proof, cosines theorem, bisector, angle.sines theorem, center of gravity, plane angle, two-sided corner

Similarities Between Trihedra and Plane Triangles

When teaching a stereometry course in academic lyceums and vocational colleges, teaching the properties of spatial bodies by relating them to the properties of plane figures known from the planimetry course helps to expand the ideas about the studied spatial bodies, the interaction of the bodies in the plane and in space. allows you to see the similarities.

For example, we see that a triangle and a tetrahedron have a number of similar properties, and in many geometrical concepts they are connected with a triangle, and we have a spatial similarity. For example: a side of a triangle and a side of a tetrahedron, an inscribed circle is an inscribed sphere, an inscribed circle is an inscribed sphere, a surface is a volume, an angle bisector is a two-sided angle bisector plane, etc.

These similarities are not only external. If we replace the planimetric terms in the formulas with their corresponding stereometric terms, most theorems about triangles become theorems about tetrahedrons. Below we will consider several such theorems and similarity relations.

First, let's look at some similarities between a trihedron and a triangle. of the trihedron α, β, γ flat corners ABC of the triangle a, b, c to the sides, A, B, C and two-sided angles of a triangle $\angle A$, $\angle B$, $\angle C$ If we match the angles, there are amazing similarities between trihedrals and triangles.

The sum of any two plane angles of a trihedron is greater than the third plane angle:

$$\beta+\gamma>\alpha$$

And in triangles, the sum of its arbitrary two sides would be greater than the third side:

$$b+c>a$$

The second theorem of cosines for trihedra is expressed as follows:

$$\cos A=-\cos B\cos C+\sin B\sin C\cos\alpha$$

$$\cos B=-\cos A\cos C+\sin A\sin C\cos\beta$$

$$\cos C=-\cos A\cos B+\sin A\sin B\cos\gamma$$

Using the above equations, we form the following equation:

$$tg\frac{A+B+C-\pi}{4}=\sqrt{tg\frac{p}{2}\cdot tg\frac{p-\alpha}{2}\cdot tg\frac{p-\beta}{2}\cdot tg\frac{p-\gamma}{2}} \qquad (*)$$

Here $p=\dfrac{\alpha+\beta+\gamma}{2}$.

Geron's formula for triangles is as follows:

$$S=\sqrt{p(p-a)(p-b)(p-c)} \qquad (**)$$

Here $p=\dfrac{a+b+c}{2}$.

(*) and (**) if we look at the equations, we can see the similarity between these equations.

The theorem of sines for trihedra is analogous to the theorem of sines for triangles:

Theorem of sines for the trihedron. The sine of the dihedral angles of a trihedron is proportional to the sine of the corresponding plane angles lying opposite them:

$$\frac{\sin A}{\sin\alpha}=\frac{\sin B}{\sin\beta}=\frac{\sin C}{\sin\gamma}.$$

Theorem of sines for triangles. The sine of the angles of an arbitrary triangle is proportional to the corresponding sides lying opposite them: $\dfrac{\sin A}{a}=\dfrac{\sin B}{b}=\dfrac{\sin C}{c}$.

Similarity between the median plane of a trihedron and the median plane of a triangle:

The plane passing through the edge forming the trihedron and the bisector of the opposite plane angle is called the median plane of this trihedron.

The median of the triangle is the median of the triangle.

The three median planes of the trihedron have a common straight line. (This straight line is called the median of the trihedron).

The three medians of a triangle intersect at one point. (This point is called the center of gravity of the triangle).

Similarity between the bisector plane of a trihedron and the bisector of a triangle:

The half-plane that departs from the edge of the trihedron and divides the two-sided angle belonging to this edge into two equal two-sided angles is called the bisector half-plane of this angle. The plane containing this half-plane is called the plane of the bisector of the two-sided angle.

The section coming out from the tip of the triangle and dividing the angle at this tip into two equal angles and connecting to the opposite side is called its bisector.

The bisector angles of a trihedron have a common straight line whose bisector planes are equal to its sides.

The angle bisectors of a triangle intersect at one point.

Similarity between elevation planes of a trihedron:

Three planes passing through each edge of the trihedron and correspondingly perpendicular to the opposite sides are called elevation planes of the trihedron.

The three sections coming out from the ends of the triangle and connecting them perpendicularly to the corresponding opposite sides are called the altitudes of this triangle.

The three elevation planes of the trihedron have a common straight line. This straight line is called the ortho axis of the trihedral.

The altitudes of the triangle intersect at one point.

If the ortho axis of the triad lies inside it, then the height planes of the triad have corresponding sides a_1, b_1, c_1 for a trihedron cut along the rays, these height planes consist of the bisector planes of its two sides.

If the altitudes of a triangle intersect inside it, these altitudes will be its bisector for the triangle formed by the points of intersection of these altitudes with the sides of the triangle.

Now let's look at some similarities between a tetrahedron and a triangle.

1 – theorem. ABC u of the corner C bof the sky CD the side whose bisector lies opposite it AC and BC divides into sections proportional to the sides.

Proof: Suppose ADC and DBC the bases of the triangles respectively AC and BC be sections (Fig. 1). Point D is equidistant from the sides of the angle ACB. that is why $\frac{S_{ADC}}{S_{BDC}}=\frac{|AC|}{|BC|}$.

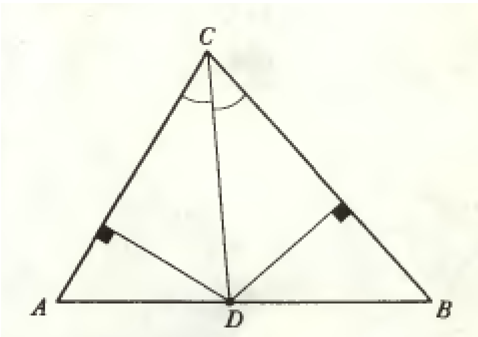

Figure 1.

Now, if we assume that the bases of these triangles are the segments AD and DB, we have the following:

$$\frac{S_{ADC}}{S_{BDC}}=\frac{|AD|}{|BD|}.$$

From this $\frac{|AD|}{|BD|}=\frac{|AC|}{|BC|}$.

2 – theorem. The bisector plane of an arbitrary edge of a tetrahedron divides the opposite edge and any one of the sides lying on this edge into pieces in such a way that the ratio of the divided edges is equal to the ratio of the surfaces of the divided sides on which they lie.

Proof. ABCD let the intersection of the tetrahedron with the two-sided angle bisector plane on the edge AD be ADM (Fig. 2). ACMD and ABMD tetrahedron volumes respectively V_1 and V_2 we define with . If we take M as the point equidistant from the sides ADC and ADB, we get the following:

$$\frac{V_1}{V_2}=\frac{S_{ADC}}{S_{ADB}}.$$

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On the other hand $\frac{V_1}{V_2}=\frac{S_{DMC}}{S_{DMB}}=\frac{|MC|}{|MB|}$.

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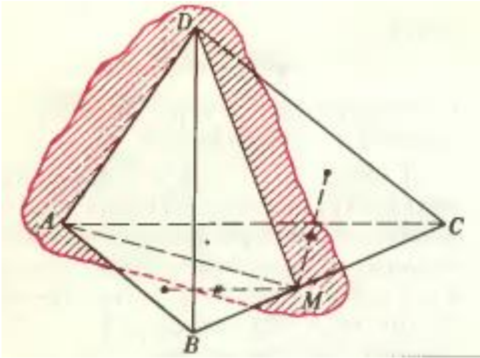


Figure 2.

Now we present the theorem about the point of intersection of triangle medians and the analogous theorem for the tetrahedron.

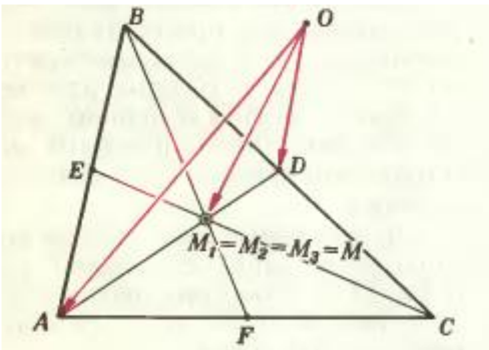
3 – theorem. The medians of a triangle intersect at one point and are divided in the ratio 2:1 at the point of intersection, starting from the tip of the triangle.

Proof: M_1 point ABC is the point from the median AD of the triangle and $|AM_1|:|M_1D| = 2:1$ and point O is an arbitrary point in space (Fig. 3)

In that case $\overrightarrow{OM_1}=\frac{1}{3}\cdot\overrightarrow{OA}+\frac{2}{3}\cdot\overrightarrow{OD}$ and $\overrightarrow{OD}=\frac{1}{2}(\overrightarrow{OB}+\overrightarrow{OC})$ will be. From this $\overrightarrow{OM_1}=\frac{1}{3}\overrightarrow{OA}+\frac{2}{3}\cdot\frac{1}{2}(\overrightarrow{OB}+\overrightarrow{OC})=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})$ our child with equality.

If M_2 and M_3 similar to the above if the points are the points taken from the medians CE and BF respectively and the length of the median is in the ratio 2:1 from the tip of the triangle

$\overrightarrow{OM_2}=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}),\quad \overrightarrow{OM_3}=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})$.



3-rasm.

From this, $\overrightarrow{OM_1}=\overrightarrow{OM_2}=\overrightarrow{OM_3}$ and M_1, M_2, M_3 the points fall on top of each other. The theorem is proved.

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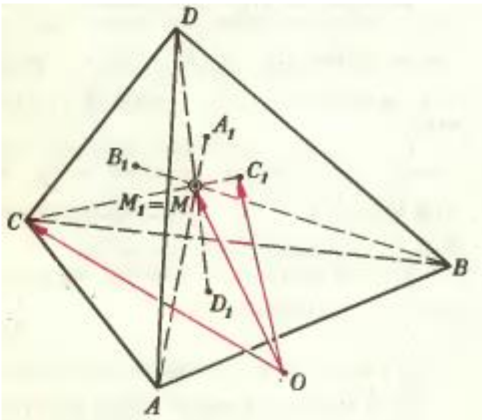
Note: The median of a tetrahedron refers to the cross section connecting the center of gravity of the opposite side to the edge of the tetrahedron.

4 – theorem. The four medians of the tetrahedron intersect at a single point and are divided in the ratio 3:1 starting from the corner tip.

Proof .M_1 point ABCD of the tetrahedron CC_1 the point taken at the median and $|CM_1|:|M_1C| = 3:1$ be the point where the condition is fulfilled (Fig. 4). We take an arbitrary point O in space. in that case

$\overrightarrow{OM_1} = \frac{1}{4}\cdot\overrightarrow{OC} + \frac{3}{4}\cdot\overrightarrow{OC_1}$ and besides

$\overrightarrow{OC_1} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OD})$ we will have equality here C_1 – ABD the center of gravity of the triangle. That is why, $\overrightarrow{OM_1} = \frac{1}{4}\cdot\overrightarrow{OC} + \frac{3}{4}\cdot\frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OD}) = \frac{1}{4}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD})$.



4-rasm.

Just as well, M_2, M_3, M_4 points respectively of the tetrahedron AA_1, BB_1, and DD_1 Let the points obtained on the medians and the lengths of the medians at these points be divided in the ratio 3:1. Considering the above,

$\overrightarrow{OM_2} = \overrightarrow{OM_3} = \overrightarrow{OM_4} = \frac{1}{4}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD})$ we have the equality and it follows that M_1, M_2, M_3, M_4 dots overlap.

5 – theorem. We take an arbitrary point O inside the triangle ABC and draw straight lines parallel to the sides of the triangle from this point. Faces of small triangles formed in agar – S_1, S_2, S_3 va ABC and if we mark the face of the triangle with S (Fig. 5), the following equality is appropriate

$\sqrt{S} = \sqrt{S_1} + \sqrt{S_2} + \sqrt{S_3}$.

Figure 5.

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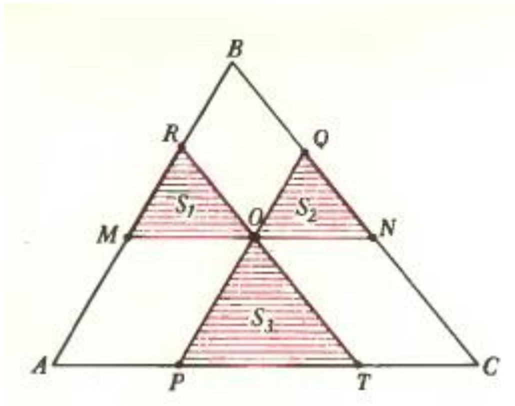
Proof. The resulting triangles are similar to triangle ABC. That is why, $\frac{\sqrt{S_1}}{\sqrt{S}}=\frac{|MR|}{|AB|}, \frac{\sqrt{S_2}}{\sqrt{S}}=\frac{|OQ|}{|AB|}, \frac{\sqrt{S_3}}{\sqrt{S}}=\frac{|OP|}{|AB|}$.

Adding these equalities, we get:



$$\frac{\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}}{\sqrt{S}}=\frac{|MR|+|OQ|+|OP|}{|AB|}=\frac{|MR|+|BR|+|MA|}{|AB|}=1$$

$$\sqrt{S}=\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}\ .$$

Now we present the analogue of the above theorem for the tetrahedron.

6 – theorem. From an arbitrary point inside the tetrahedron, we draw four planes parallel to the edges of the tetrahedron. The volumes of small tetrahedra formed in agar are V_1, V_2, V_3, V_4 and if the volume of the given tetrahedron is V, the following equality is appropriate

$$\sqrt[3]{V}=\sqrt[3]{V_1}+\sqrt[3]{V_2}+\sqrt[3]{V_3}+\sqrt[3]{V_4}.$$

In addition, we can give examples of the following analogies.

a – part is a theorem in planimetry, part b is an analogue in space.

1 – issue. a) Face of a triangle S and the following formula is appropriate for the circle inscribed in it $S=\frac{1}{2}pr$, where p is the perimeter of the triangle , r – the radius of the inscribed circle.

The following formula is appropriate for the volume of the tetrahedron V and the sphere drawn inside it $V=\frac{1}{3}Sr$, where S is the total surface of the tetrahedron, r is the radius of the sphere.

2 – issue. a) If the radius of the circle inscribed in the triangle is r, equality is appropriate here h_1, h_2, h_3 – triangle heights.

If the radius of the sphere inscribed in the tetrahedron is r , $\frac{1}{r}=\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}+\frac{1}{h_4}$ equality is appropriate here h_1, h_2, h_3, h_4 – the heights of the tetrahedron.

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