REPLACING THE MULTIPLIER IN SOLVING EXPONENTIAL INEQUALITIES USING THE METHOD

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ABSTRACT

This article provides theoretical information about one of the methods of solving exponential inequalities, the method of replacing the multiplier. Equivalency conditions for the multiplier replacement method are given and examples are solved.

Keywords: Inequality, multiplier, equipotent, canonical representation, homogeneous polynomial, solution, method.

There are several ways to solve exponential inequalities. In this article, we will describe one of the ways to solve it, the method of replacing the multiplier.

Equal strength conditions for the multiplier replacement method

1. $\begin{cases} a^x \vee b, \\ b>0 \end{cases} \Leftrightarrow \begin{cases} a>1, \\ x \vee \log_a b \end{cases} \cup \begin{cases} a\in(0;1) \\ x \wedge \log_a b \end{cases} \Leftrightarrow \begin{cases} a-1>0, \\ x-\log_a b \vee 0 \end{cases} \cup \begin{cases} a-1<0, \\ -(x-\log_a b) \vee 0 \end{cases}$

Summary: $\begin{cases} a^x - b \vee 0, \\ b>0 \end{cases} \Leftrightarrow (a-1)(x-\log_a b) \vee 0$

2. $\begin{cases} a^x - b < 0, \\ b \le 0 \end{cases} \Leftrightarrow x \in \varnothing$, because $\forall x \in R$ da $a^x > 0$.

3. $\begin{cases} a^x - b > 0, \\ b \le 0 \end{cases} \Leftrightarrow x \in R$

4. $a^{f(x)} \vee a^{g(x)} \Leftrightarrow \begin{cases} a>1, \\ f(x) \vee g(x) \end{cases} \cup \begin{cases} a\in(0;1), \\ f(x) \wedge g(x) \end{cases} \Leftrightarrow \begin{cases} a-1>0, \\ f(x)-g(x) \vee 0 \end{cases} \cup \begin{cases} a-1<0, a>0 \\ -(f(x)-g(x)) \vee 0 \end{cases}$

Summary: $a^{f(x)} - a^{g(x)} \vee 0 \Leftrightarrow (a-1)(f(x)-g(x)) \vee 0$

Special cases.

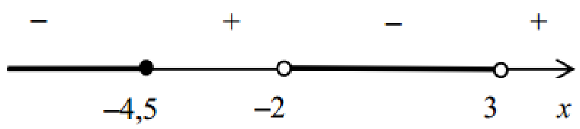
1. $\begin{cases} a^{f(x)} - b \vee 0 \\ b>0 \end{cases} \Leftrightarrow a^{f(x)} - a^{\log_a b} \vee 0 \Leftrightarrow (a-1)(f(x)-\log_a b) \vee 0.$

2. $a^{f(x)} - 1 \vee 0 \Leftrightarrow a^{f(x)} - a^0 \Leftrightarrow (a-1) f(x) \vee 0.$

3. Special cases. $7^{\frac{1}{x+2}} \ge (\frac{1}{7})^{\frac{3}{3-x}}$ solve the inequality.

4. 4. Solving. We use the method of replacing the multiplier.

$$7^{\frac{1}{x+2}} - 7^{\frac{3}{3-x}} \ge 0 \Leftrightarrow (7-1)(\frac{1}{x+2} - \frac{3}{x-3}) \ge 0 \Leftrightarrow \frac{x-3-3x-6}{(x+2)(x-3)} \ge 0 \Leftrightarrow \frac{2x+9}{(x+2)(x-3)} \le 0$$



$x \in (-\infty; -4,5] \cup (-2;3)$

Answer: $x \in (-\infty; -4,5] \cup (-2; 3)$

Example 2. This $\sqrt[3]{2^{x^2-6x-4}} \geq (\sqrt{3+\sqrt{8}}-1)^x$ solve the inequality

Solving.

1) $3+\sqrt{8} = 3+2\sqrt{2} = (\sqrt{2}+1)^2$

2) $(\sqrt{3+\sqrt{8}}-1)^x = (\sqrt{(\sqrt{2}+1)^2}-1)^x = (\sqrt{2}+1-1)^x = (\sqrt{2})^x = 2^{\frac{x}{2}}$

3) $2^{\frac{x^2-6x-4}{3}} - 2^{\frac{x}{2}} \geq 0 \Leftrightarrow (2-1)(\frac{x^2-6x-4}{3} - \frac{x}{2}) \geq 0 \Leftrightarrow 2x^2 - 12x - 8 - 3x \geq 0 \Leftrightarrow$

$2x^2 - 15x - 8 \geq 0 \Leftrightarrow (2x+1)(x-8) \geq 0 \Leftrightarrow x \in (-\infty; -0,5] \cup [8; \infty)$

Answer: $x \in (-\infty; -0,5] \cup [8; \infty)$

Example 3. This $5^{x+2} + 5^{-x} - 23 \geq \log_4 64$ solve the inequality.

Solving. We convert the given inequality into canonical form and use the method of substitution of multipliers.

$25 \cdot 5^x + 5^{-x} - 23 - \log_4 4^3 \geq 0 \Leftrightarrow 25 \cdot 5^x + 5^{-x} - 26 \geq 0$

$t = 5^x, t > 0$ let it be

$$\begin{cases} 25t + \frac{1}{t} - 26 \geq 0, \\ t > 0 \end{cases} \Leftrightarrow \begin{cases} 25t^2 - 26t + 1 \geq 0, \\ t > 0 \end{cases} \Leftrightarrow \begin{cases} (25t-1)(t-1) \geq 0, \\ t > 0 \end{cases} \Leftrightarrow \begin{cases} (25 \cdot 5^x - 1)(5^x - 1) \geq 0, \\ 5^x > 0 \end{cases}$$

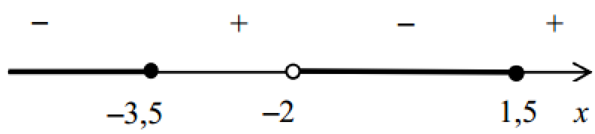
$\Leftrightarrow (5^{x+2} - 5^0)(5^x - 5^0) \geq 0 \Leftrightarrow (5-1)(x+2-0)(5-1)(x-0) \geq 0 \Leftrightarrow (x+2)x \geq 0 \quad x \in (-\infty; -2] \cup [0; \infty)$

Answer: $x \in (-\infty; -2] \cup [0; \infty)$

Example 4. This $\dfrac{9^{x^2+5x-6} - \left(\frac{1}{3}\right)^{2x^2-2x-9}}{2^{3x-4} - (0,5)^{6-2x}} \leq 0$ solve the inequality

Solving. $\dfrac{3^{2x^2+10x-12} - 3^{-2x^2+2x+9}}{2^{3x-4} - 2^{2x-6}} \leq 0$ We use the method of replacing the multiplier.

$$\frac{(3-1)(2x^2+10x-12-(-2x^2+2x+9))}{(2-1)((3x-4)-(2x-6))} \leq 0 \Leftrightarrow \frac{4x^2+8x-21}{x+2} \leq 0 \Leftrightarrow \frac{(2x+7)(2x-3)}{x+2} \leq 0$$



$x \in (-\infty; -3,5] \cup (-2; 1,5]$

Answer: $x \in (-\infty; -3,5] \cup (-2; 1,5]$

Example 5. This $\dfrac{5 \cdot 4^x - 6 - 7 \cdot 10^x + 4 \cdot 25^x}{25^x - 3} \leq 2$ solve the inequality.

Solving.

1) We make the inequality in its canonical form and use the method of replacing the multiplier.

2) $$\frac{5\cdot 4^x-6-7\cdot 10^x+4\cdot 25^x-2\cdot 25^x+6}{25^x-3}\le 0 \Leftrightarrow \frac{2\cdot 25^x-7\cdot 10^x+5\cdot 4^x}{25^x-3}\le 0 \Leftrightarrow \frac{f(x)}{g(x)}\le 0,$$

in this $f(x)=2\cdot 25^x-7\cdot 10^x+5\cdot 4^x$, $g(x)=25^x-3$

2) $f(x)$ and $g(x)$ we change the function pointers to the same functions

a) $f(x)\vee 0 \Leftrightarrow 2\cdot 5^{2x}-7\cdot 5^x\cdot 2^x+5\cdot 2^{2x}\vee 0$ (*)

$f(x)$ 5^x and 2^x funksiyalarga nisbatan bir jinsli ikkinchi darajali ko'pha Homogeneous quadratic polynomial with respect to functions

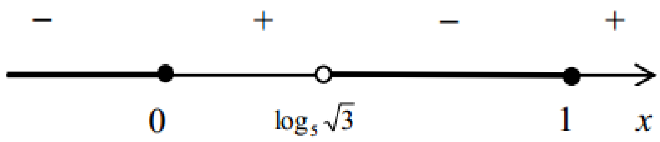
d. $4^x>0, \forall x\in R$ for being (*) inequality 4^x we will be

$$2\cdot(\frac{5}{2})^{2x}-7\cdot(\frac{5}{2})^x+5\vee 0 \Leftrightarrow 2((\frac{5}{2})^x-1)((\frac{5}{2})^x-\frac{5}{2})\vee 0 \Leftrightarrow$$

$$((\frac{5}{2})^x-(\frac{5}{2})^0)\ ((\frac{5}{2})^x-\frac{5}{2})\vee 0 \Leftrightarrow (\frac{5}{2}-1)(x-0)(\frac{5}{2}-1)(x-1)\vee 0 \Leftrightarrow x(x-1)\vee 0.$$

b) $g(x)\vee 0 \Leftrightarrow 5^{2x}-5^{\log_5\sqrt{3}}\vee 0 \Leftrightarrow (5-1)(2x-\log_5 3)\vee 0 \Leftrightarrow x-\frac{\log_5 3}{2}\vee 0 \Leftrightarrow x-\log_5\sqrt{3}\vee 0.$

3) So, $\frac{f(x)}{g(x)}=\frac{x(x-1)}{x-\log_5\sqrt{3}}\le 0$



$x\in(-\infty;0]\cup(\log_5\sqrt{3};1]$

Answer: $x\in(-\infty;0]\cup(\log_5\sqrt{3};1]$

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