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SOLUTION OF THE BOUNDARY VALUE PROBLEM OF A SEMI-INFINITE WAVEGUIDE

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ANNOTATION

In this work, more accurate but more complex methods are proposed for propagation of oscillations in cylindrical waveguides. This problem is considered in relation to a specific technical problem of the propagation of elastic vibrations along the drill string arising from a bit working at the bottom of a native well. Information from the signals received at the wellhead in the string can serve as valuable information about the operation of engines at the bottomhole, about the rock near the bottomhole, etc.

Keywords: Matrix, bar, inclusions, plastic zone, edge effects, stresses, deformation, crack, trivial, dissipation.

INTRODUCTION

Consider a semi-infinite casing that occupies an area $0<z<\infty$. Let at the initial moment of time $t = 0$ she was at rest. At the edge of the pipe at $z = 0$ a given external disturbance (for example, from the source of vibration of the drilling tool) acts [1], which can be described as follows:

$$\omega = g_1(t), \qquad \frac{\partial\omega}{\partial t} = g_2(t) \quad \text{при} \quad z = 0 \;, \qquad (1)$$

here $g_1(t)$ and $g_2(t)$ – заданные функции.

Boundary value problem (1) will be called the first main problem [2-3].

At infinity at $z \to\infty$, the perturbation should decay, i.e.

$$\omega = 0, \qquad \frac{\partial\omega}{\partial t} = 0 \quad \text{at} \quad z \to\infty \;. \qquad (2)$$

Note that an even continuation of the function $\omega(z,t)$ to the region $-\infty <z< 0$, which is a solution to the first basic problem, will also satisfy the basic differential equation. Therefore, the constructed even function will be a solution to problem (1) for an infinite casing string.

Initial conditions are zero - $\omega = \dfrac{\partial\omega}{\partial t} = 0$ at $t = 0$. (3)

First, we represent the functions $g_1(t)$ and $g_2(t)$ as follows [4]

$$g_1(t) = \int_0^\infty \delta(t-\tau)g_1(\tau)d\tau, \qquad g_2(t) = \int_0^\infty \delta(t-\tau)g_2(\tau)d\tau \;, \qquad (4)$$

where $\delta(\xi)$ is a delta function.

Since the considered problem (1) is linear, at first one can find a simpler solution for the analytical analysis of the problem

$$\omega = \delta(t-\tau), \quad \frac{\partial \omega}{\partial t} = \delta(t-\tau) \quad \text{at} \quad z = 0, \tag{5}$$

where τ is some arbitrary parameter.

Then, from the obtained solution by superposition with respect to the parameter using formulas similar to (4), one can construct a general analytical solution of the first basic problem in the form of the following integral:

$$\omega(z,t) = \int_0^t \omega_1(z,t-\tau) g_1(\tau)\, d\tau + \int_0^t \omega_2(z,t-\tau) g_2(\tau)\, d\tau, \tag{6}$$

here $\omega_1(z,t)$and $\omega_2(z,t)$ are particular solutions of the first main problem satisfying the conditions

1) $\quad \omega_1 = \delta(t), \quad \frac{\partial \omega}{\partial t} = 0 \quad$ at $\quad z=0$, $\qquad$ (7)

2) $\quad \omega_2 = 0, \quad \frac{\partial \omega}{\partial t} = \delta(t) \quad$ at $\quad z=0$, $\qquad$ (8)

3) $\quad \omega_1 = \omega_2 = 0 \quad$ at $\quad t < 0$. $\qquad$ (9)

We multiply both sides of the basic differential equation by e^{-pt}, where P is a complex parameter, and integrate over t from zero to infinity, that is, we perform Laplace transformations. We denote

$$\overline{\omega}(z,p) = \int_0^\infty e^{-pt} \omega(z,t)\, dt \quad . \tag{10}$$

Let's perform the following calculations

$$\int_0^\infty \frac{\partial \omega}{\partial t} e^{-pt} dt = \int_0^\infty e^{-pt} d\omega(t) = e^{-pt}\omega \Big|_0^\infty -$$

(transformation of integral by parts)

$$-(-P)\int_0^\infty \omega p^{-pt} dt = p\overline{\omega}(z,p) \ , \tag{11}$$

$$\int_0^\infty \frac{\partial^2 \omega}{\partial t^2} e^{-pt} dt = e^{-pt}\frac{\partial \omega}{\partial t} + P\int_0^\infty \frac{\partial \omega}{\partial t} e^{-pt} dt =$$

(transformation of integral by parts)

$$= e^{-pt}\frac{\partial \omega}{\partial t}\Big|_0^\infty + Pe^{-pt}\omega\Big|_0^\infty + P^2 \omega e^{-pt} dt = P^2 \overline{\omega}(z,p) \ . \tag{12}$$

The following conditions were used here

$$W = \frac{\partial \omega}{\partial t} = 0, \quad \text{at} \quad t=0 \ . \tag{13}$$

For $t \to \infty$ we have

$$e^{-pt}\,\omega \to 0, \qquad e^{-pt}\frac{\partial \omega}{\partial t} \to 0, \tag{14}$$

since Rep >0 an ω and $\frac{\partial \omega}{\partial t}$ are bounded.

The main differential equation is reduced to the form

$$E_0 J \frac{d^4\overline{\omega}}{dt^4} = -\left(\alpha p^2 + \eta p + K_\lambda\right)\overline{\omega}\,. \tag{15}$$

The general solution of this equation has the form

$$\overline{\omega}(z,p) = C_1 e^{\lambda_1 z} + C_2 e^{\lambda_2 z} + C_3 e^{\lambda_3 z} + C_4 e^{\lambda_4 z}\;, \tag{16}$$

where C_1, C_2, C_3, C_4 are arbitrary constants that may depend on the parameter P; λ_1, λ_2, λ_3, λ_4 are different roots of the equation

$$\lambda^4 = -\frac{1}{E_0 J}\left(\alpha p^2 + \eta p + K_\lambda\right). \tag{17}$$

Thus, these roots are functions of the complex parameter P.

Now we apply the Laplace transform to the boundary conditions (7) and (8), respectively, we obtain

$$1)\quad \overline{\omega}_1 = 1, \quad \frac{d_1\overline{\omega}}{dz} = 0 \quad \text{at} \quad z=0\,, \tag{18}$$

$$2)\quad \overline{\omega}_2 = \frac{1}{p}, \quad \frac{d\overline{\omega}_2}{dz} = 0 \quad \text{at } z=0\,. \tag{19}$$

The last conditions in these formulas are the particular conditions for the functions ω_1 and ω_2 with respect to z.

Let us analyze equation (17), denoting its right-hand side as follows:

$$\mathrm{Re}^{i\varphi} = -\frac{1}{E_0 J}\left(\alpha p^2 + \eta p + K_\lambda\right), \tag{20}$$

the roots of equation (17) can be written in the form

$$\lambda_1 = R^{1/4} e^{i\frac{\varphi}{4}}, \quad \lambda_2 = R^{1/4}\exp i\left(\frac{\varphi}{4} + \frac{\pi}{2}\right), \tag{21}$$

$$\lambda_3 = R^{1/4}\exp i\left(\frac{\varphi}{4} + \pi\right), \qquad \lambda_4 = R^{1/4}\exp i\left(\frac{\varphi}{4} + \frac{3\pi}{2}\right). \tag{22}$$

Two of these roots always have a positive real part, and the other two have a negative part. For example, let the roots λ_3 and λ_4 have a positive real part (this can always be achieved by renaming the roots in (22). According to condition (2), at infinity for $z \to \infty$ and formula (10), the condition

$$\overline{\omega}\,(Z, P) = 0 \quad \text{at} \quad z \to \infty\,. \tag{23}$$

Therefore, in order for this condition to be fulfilled, it is necessary in the general solution (16) to put

$$C_3 = C_4 = 0\,. \tag{24}$$

Thus,

$$\overline{\omega} = C_1 e^{\lambda_1 z} + C_2 e^{\lambda_2 z}, \qquad \frac{d\omega}{dt} = C_1\lambda_1 e^{\lambda_1 z} + C_2\lambda_2 e^{\lambda_2 z}. \tag{25}$$

We now use conditions (18) and (19):

1. ω_1: $C_1+C_2 = 1$, $C_1\lambda_1 + C_2\lambda_2 = 0$, (26)
2. ω_2: $C_1+C_2 = \frac{1}{P}$, $C_1\lambda_1 + C_2\lambda_2 = 0$. (27)

Solving these equations, we find:

1. ω_1: $C_1 = \lambda_2/(\lambda_2 - \lambda_1)$, $C_2 = \lambda_1/(\lambda_1 - \lambda_2)$, (28)
2. ω_2: $C_1 = \frac{\lambda}{P}(\lambda_2 - \lambda_1)$, $C_2 = \lambda_1/p(\lambda_1 - \lambda_2)$. (29)

The Laplace transforms of the sought-for particular solutions ω_1 and ω_2 can be written as

$$\overline{\omega}_1(z,p) = \frac{1}{\lambda_2 - \lambda_1}\left(\lambda_2 e^{\lambda_1 z} - \lambda_1 e^{\lambda_2 z}\right), \tag{30}$$

$$\overline{\omega}_2(z,p) = \frac{1}{p(\lambda_2 - \lambda_1)}\left(\lambda_2 e^{\lambda_1 z} - \lambda_1 e^{\lambda_2 z}\right) \quad . \tag{31}$$

Using the inverse Laplace transform, we find the particular solutions $W_1(z,t)$ and $W_2(z,t)$ themselves:

$$\omega_1(z,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{e^{pt}}{\lambda_2 - \lambda_1}\left(\lambda_2 e^{\lambda_1 z} - \lambda_1 e^{\lambda_2 z}\right)dp, \tag{32}$$

$$\omega_2(z,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{e^{pt}}{p(\lambda_2 - \lambda_1)}\left(\lambda_2 e^{\lambda_1 z} - \lambda_1 e^{\lambda_2 z}\right)dp, \tag{33}$$

here C is some number greater than zero.

An analytical solution of the no stationary boundary value problem of oscillations of a semi-infinite waveguide is given by the method of the integral Laplace transform [5].

LITERATURE

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