EXPANSIONS OF HYPERGIOMETRIC FUNCTIONS OF SEVERAL VARIABLES ACCORDING TO KNOWN FORMULAS

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ANNOTATION

This article presents some key facts. Hypergeometric functions of several variables and Burchnell-Chandy operators necessary to present the main results of the paper are presented.

Keywords: Gorn list; complete and confluent hypergeometric functions of the second order; Srivastava-Karlsson list; confluent hypergeometric functions in three variables.

The relationship between these two systems is only in the case of two variables
well studied. Even in classical Horn, Appell, Poxgammer and Lauricella, multivariate hypergeometric functions, only in the 1970s and 80s, there was an attempt to study the Lie algebra of differential equations by W. Miller and his students. In the last three decades, there has been a growing interest in the study of hypergeometric functions. In fact, a search of the database for the term hypergeometric yields 3,181 articles, 1,530 of which have been published since 1990. This new interest stems from the connection between hypergeometric functions and many areas of mathematics, such as algebraic geometry. , combinatorics, number theory, symmetric reflections, etc.

It is known that the Gamma function $\Gamma(s)$ is defined by the following integral:

$$\Gamma(s)=\int_0^{\infty} e^{-t}t^{x-1}dt \qquad (1)$$

(1) $\mathrm{Re}(s)>0$ represents a holomorphic function on the integral half-plane, and hence except that it satisfies the following functional equation.

$$\Gamma(s+1)=s\Gamma(s);\quad \mathrm{Re}(s)>0 \qquad (2)$$

Hence, $\Gamma(1)=1$ it follows that $\Gamma(n+1)=n!,(\,)$ is $.\, n\in N$

$\Gamma(s)$ We can use (2) to expand to a meromorphic function on the entire complex plane with simple poles in non-positive integers.

For example, we define $\{-1<\mathrm{Re}(s)\leq 0\ \}$ palasada $\Gamma(s)$ as follows:

$$\Gamma(s)=\frac{\Gamma(s+1);}{s}$$

Description . $\alpha\in C/Z_{\leq 0}$ and $k\in N$ given that we have the Poxgammer symbol we define:

$$(\alpha)_k = \frac{\Gamma(a+k);}{\Gamma(a)} \qquad (3)$$

Let's say $n=(n_1,n_2,......n_r)\in N^r$ r-order of non-negative integers

let it be Just like that $x=(x_1,x_2,......x_r)\in C^r$ if given, we do it

x^n we define with .

In that case

$x^n = (x_1^{n_1}, x^{n_2}{}_2, ...x^{n_r}{}_r)$ and Q^r the standard basis vector of order e_j j in we define with .

Description. All $j=1,2,........,r$ for the

$R_j(n) = \frac{A_{n+e_j}}{A_n}$ the ratio $n=(n_1,n_2,......n_r)$ of this multivariate rank series

$$F(x_1,x_2,.....,x_r) = \sum_{n\in N^r}^{\infty} A_n x^n$$

It is called the Gorn hypergeometric function.

The following series are commonly called Gaussian hypergeometric series

$${}_2F_1(\alpha,\beta,\gamma;x) = \sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{(\gamma)_n n!}x^n; \gamma\in Z_{\le 0} \qquad (4)$$

The recurrent properties of the coefficients of the hypergeometric series allow us to express the solution of their ordinary or partial differential equations.

that the Gaussian hypergeometric function produces a second-order ordinary differential equation satisfied by $\frac{d}{dx}$ differentiation operator ∂_x through, for functions of $\frac{\partial}{\partial x_j}$ several variables we $(x_1,x_2,......x_r)$ for the private derivative operator

∂_j we use . Or, for simplicity, we can use the following Euler operators:

$$\theta_x = x\partial_x; \theta_j = x_j\partial_j$$

Now consider Gaussian hypergeometric series (4). It is known that

$$\theta_x F(\alpha,\beta,\gamma;x) = \sum_{m,n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{(\gamma)_n n!}x^n;$$

But, $n(\alpha)_n = \alpha((\alpha+1)-(\alpha)_n)$ in accordance with

$$\theta_x F(\alpha,\beta,\gamma;x) = \alpha\sum_{m,n=0}^{\infty}\left(\frac{(\alpha+1)_n(\beta)_n}{(\gamma)_n n!} - \frac{(\alpha)_n(\beta)_n}{(\gamma)_n n!}\right)x^n = \alpha\left(F(\alpha+1,\beta,\gamma;x) - F(\alpha,\beta,\gamma;x)\right).$$

So,

$$\left(\theta_x+\alpha\right)F(\alpha,\beta,\gamma;x)=\alpha F(\alpha+1,\beta,\gamma;x)$$

$$\left(\theta_x+\beta\right)F(\alpha,\beta,\gamma;x)=\alpha F(\alpha,\beta+1,\gamma;x)$$

Similarly, we have the following equality:

$$\left(\theta_x+(\gamma-1)\right)F(\alpha,\beta,\gamma;x)=\left(\gamma-1\right)F(\alpha,\beta,\gamma-1;x)$$

$$\partial_x F(\alpha,\beta,\gamma;x)=\frac{\alpha\beta}{\gamma}F(\alpha+1,\beta+1,\gamma+1;x)$$

Gaussian hypergeometric series by combining the above four equations
we can find that satisfies the following ordinary differential equation:

$$\left(\theta_x+\alpha\right)\left(\theta_x+\beta\right)F=(\theta_x+\gamma)\partial_x F \qquad (5)$$

We can see that equation (5) is equivalent to this equation:

$$x(x-1)\partial_x^2 F+((\alpha+\beta+1)x-\gamma)\partial_x F+\alpha\beta F=0$$

In this way, through hypergeometric functions and certain formulas the mysteries of diffusion as functions of several variables have been explored and are still being explored.

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