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LINEAR CONTROLLER DESIGN OF A NONLINEAR CONTROL SYSTEM GUARANTEEING GLOBAL EXPONENTIAL STABILITY

Yeong-Jeu Sun
Professor of Department of Electrical Engineering, I-Shou University, Kaohsiung, Taiwan
Tel: 886-7-6577711 ext. 6626; Fax: 886-7-6577205; Email: yjsun@isu.edu.tw

Che-Ming Chuang
Postgraduate Student of Department of Electrical Engineering, I-Shou University

Ting-Chia Chang
Postgraduate Student of Department of Electrical Engineering, I-Shou University

Yu-Chi Ho
Postgraduate Student of Department of Electrical Engineering, I-Shou University

ANNOTATION

This paper is aimed at a nonlinear system, and intends to design a simple linear controller, so that the entire closed-loop control system can achieve the goal of global exponential stability. Besides, the exponential convergence rate of such a system will also be rigorously derived. Finally, we will provide a numerical simulation example to verify the correctness and applicability of the main theorem in this paper.

Keywords: global exponential stability, nonlinear systems, exponential convergence rate, linear control.

INTRODUCTION

As we know, real physical systems are nonlinear systems, and we often use its linear model for analysis and design, mainly because the analysis and design of linear systems are easier than nonlinear systems. However, when the design results are applied to real nonlinear systems, distortion or biased results often occur. Therefore, it is reasonable and correct way to analyze and design the actual nonlinear system. In the past, there have been some well-developed methodologies for controller design of various nonlinear systems; such as sliding mode control methodology, differential and integral inequalities, backstepping approach, and others; see, for example, [1-7] and the references therein. This paper will focus on a nonlinear system, using the methodology of differential and integral inequalities, to design a linear controller that is easy to implement in hardware, so that the closed-loop control system can achieve the goal of global exponential stability.

PROBLEM FORMULATION AND MAIN RESULTS

This paper explores the following fifth-order nonlinear dynamical system:

$$\dot{x}_1 = -c_1 x_1 - c_1 c_2 x_2 + c_1 x_3 + u_1, \tag{1a}$$

$$\dot{x}_2 = c_1 c_2 x_1 - c_1 x_2 + c_1 x_4 + u_2, \tag{1b}$$

$\dot{x}_3 = c_3 x_1 - x_3 + c_2 x_4 - x_1 x_5 + u_3,$ (1c)

$\dot{x}_4 = c_3 x_2 - c_2 x_3 - x_4 - x_2 x_5 + u_4,$ (1d)

$\dot{x}_5 = -c_4 x_5 + x_1 x_3 + x_2 x_4, \ \forall t \ge 0,$ (1e)

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t) \quad x_5(t)]^T \in \Re^{5\times 1}$ is the state vector, $u(t) := [u_1(t) \quad u_2(t) \quad u_3(t) \quad u_4(t)]^T \in \Re^{4\times 1}$ is the system control, $[x_{10} \quad x_{20} \quad x_{30} \quad x_{40} \quad x_{50}]^T$ is the initial value, and c_1, c_2, c_3, c_4 are the parameters of the system (1), with $c_1 > 0$ and $c_4 > 0$. In case of $c_1 = 2, c_2 = 0.002, c_3 = 20, c_4 = 0.25$, and $u(t) = 0$, the above system is a well-known laser dynamic system [8-9], and chaos will occur in such a system.

The global exponential stabilization of system (1) and its exponential convergence rate are respectively defined as follows.

Definition 1. The system (1) is said to be globally exponentially stable if there exist a control u and positive number α satisfying

$\|x(t)\| \le \|x(0)\| \cdot e^{-\alpha t}, \quad \forall t \ge 0.$

At the same time, the parameter α is called the exponential convergence rate.

The purpose of this paper is to design a simple linear control such that the global exponential stabilization of the system (1) can be achieved. In addition, we also explore the exponential convergence rate of this stable system at the same time.

In the following, we propose the main result for the globally exponential stabilization of nonlinear system (1) by using differential and integral inequalities.

Theorem 1. The nonlinear system (1) realizes the globally exponential stabilization under the linear control

$$u_1 = \left(c_1 - c_4 - \frac{|c_1| + |c_3|}{2} \right) x_1, \tag{2a}$$

$$u_2 = \left(c_1 - c_4 - \frac{|c_1| + |c_3|}{2} \right) x_2, \tag{2b}$$

$$u_3 = \left(1 - c_4 - \frac{|c_1| + |c_3|}{2} \right) x_3, \tag{2c}$$

$$u_4 = \left(1 - c_4 - \frac{|c_1| + |c_3|}{2} \right) x_4. \tag{2d}$$

At the same time, the guaranteed exponential convergence rate is given by

$$\alpha := c_4. \tag{3}$$

Proof. Let

$$W(x(t)) := \frac{x_1^2(t)}{2} + \frac{x_2^2(t)}{2} + \frac{x_3^2(t)}{2} + \frac{x_4^2(t)}{2}. \tag{4}$$

The time derivative of $W(x(t))$ along the trajectories of the closed-loop systems (1) with (2) is given by

$$\begin{aligned}
&\dot{W}(x(t)) \\
&= x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 + x_4\dot{x}_4 + x_5\dot{x}_5 \\
&= x_1\left(-c_4x_1 - c_1c_2x_2 + c_1x_3 - \frac{|c_1|+|c_3|}{2}x_1\right) + x_2\left(-c_4x_2 + c_1c_2x_1 + c_1x_4 - \frac{|c_1|+|c_3|}{2}x_2\right) \\
&\quad + x_3\left(-c_4x_3 + c_3x_1 + c_2x_4 - x_1x_5 - \frac{|c_1|+|c_3|}{2}x_3\right) \\
&\quad + x_4\left(-c_4x_4 + c_3x_2 - c_2x_3 - x_2x_5 - \frac{|c_1|+|c_3|}{2}x_4\right) + x_5(-c_4x_5) \\
&\le -c_4x_1^2 + |c_1||x_1||x_3| - \frac{|c_1|+|c_3|}{2}x_1^2 - c_4x_2^2 + |c_1||x_2||x_4| - \frac{|c_1|+|c_3|}{2}x_2^2 - c_4x_3^2 + |c_3||x_1||x_3| \\
&\quad + c_2x_3x_4 - x_1x_3x_5 - \frac{|c_1|+|c_3|}{2}x_3^2 - c_4x_4^2 + |c_3||x_2||x_4| - c_2x_3x_4 - x_2x_4x_5 - \frac{|c_1|+|c_3|}{2}x_4^2 \\
&\quad - c_4x_5^2 + x_1x_3x_5 + x_2x_4x_5 \\
&= (|c_1|+|c_3|)|x_1||x_3| + (|c_1|+|c_3|)|x_2||x_4| - \frac{|c_1|+|c_3|}{2}x_1^2 - \frac{|c_1|+|c_3|}{2}x_2^2 - \frac{|c_1|+|c_3|}{2}x_3^2 \\
&\quad - \frac{|c_1|+|c_3|}{2}x_4^2 - c_4\left(x_1^2+x_2^2+x_3^2+x_4^2+x_5^2\right) \\
&\le \frac{|c_1|+|c_3|}{2}|x_1|^2 + \frac{|c_1|+|c_3|}{2}|x_3|^2 + \frac{|c_1|+|c_3|}{2}|x_2|^2 + \frac{|c_1|+|c_3|}{2}|x_4|^2 - \frac{|c_1|+|c_3|}{2}x_1^2 - \frac{|c_1|+|c_3|}{2}x_2^2 \\
&\quad - \frac{|c_1|+|c_3|}{2}x_3^2 - \frac{|c_1|+|c_3|}{2}x_4^2 - c_4\left(x_1^2+x_2^2+x_3^2+x_4^2+x_5^2\right) \\
&= -2c_4W, \quad \forall\, t \ge 0.
\end{aligned}$$

Thus, one has

$$e^{2c_4t}\cdot\dot{W} + e^{2c_4t}\cdot 2\alpha WV = \frac{d}{dt}\left[e^{2c_4t}\cdot W\right] \le 0, \quad \forall\, t \ge 0.$$

It follows

$$\int_0^t \frac{d}{d\tau}\left[e^{2c_4\tau}\cdot W(x(t))\right]d\tau = e^{2c_4t}\cdot W(x(t)) - W(x(0)) \le \int_0^t 0\, d\tau = 0, \quad \forall\, t \ge 0. \tag{5}$$

From (4) and (5), it results

$$\|x(t)\|^2 = W(x(t)) \le e^{-2c_4t}W(x(0)) = e^{-2c_4t}\|x(0)\|^2, \quad \forall\, t \ge 0.$$

As a consequence, we conclude that

$$\|x(t)\| \le e^{-c_4t}\|x(0)\|, \quad \forall\, t \ge 0.$$

This completes the proof.

NUMERICAL SIMULATIONS

Consider the nonlinear system of (1) with

$$c_1 = 2,\ c_2 = 0.002,\ c_3 = 20,\ c_4 = 0.25. \tag{6}$$

From (2) with (6), it can be readily obtained that

$$u_1 = -9.25x_1, \tag{7a}$$

$$u_2 = -9.25x_2, \tag{7b}$$

$$u_3 = -10.25x_3, \tag{7c}$$

$$u_4 = -10.25x_4. \tag{7d}$$

As a consequence, by Theorem 1, we conclude that the nonlinear system (1) with (6) is globally exponentially stable under the linear control of (7). At the same time, from (3), the guaranteed exponential convergence rate is given by $\alpha = 0.25$. The typical state trajectories of the uncontrolled system and the feedback-controlled system are shown in Figure 1 and Figure 2, respectively. Meanwhile, the control signals and the hardware implementation diagram of the controller of (7) are shown in Figure 3 and Figure 4, respectively.

CONCLUSION

In this paper, the globally exponential stabilization of a nonlinear system has been studied. Based on the methodology of differential and integral inequalities, a linear controller has been designed to ensure that the closed-loop control system achieves the goal of global exponential stability. The controller design for a more generalized nonlinear system will be one of the future research directions of our team.

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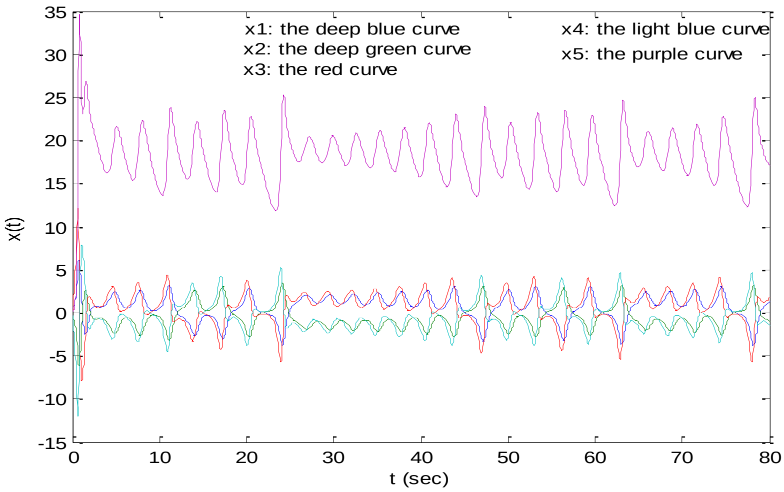

Figure 1: Typical state trajectories of the system (1) with (6) and $u = 0$.

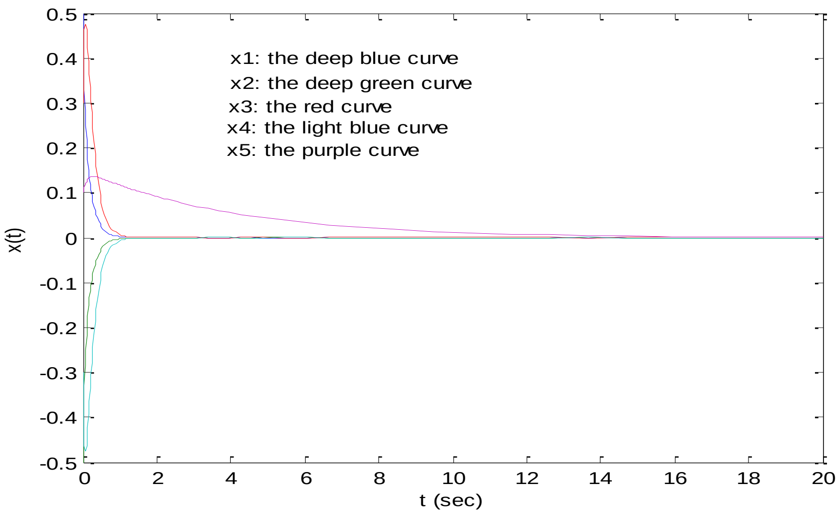

Figure 2: Typical state trajectories of the feedback-controlled system of (1) with (6) and (7).

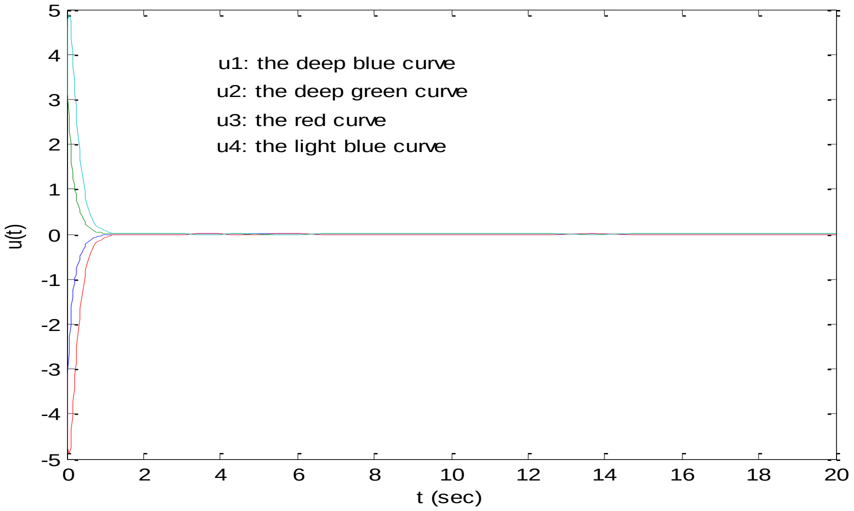

Figure 3: Control signals of $u(t)$.

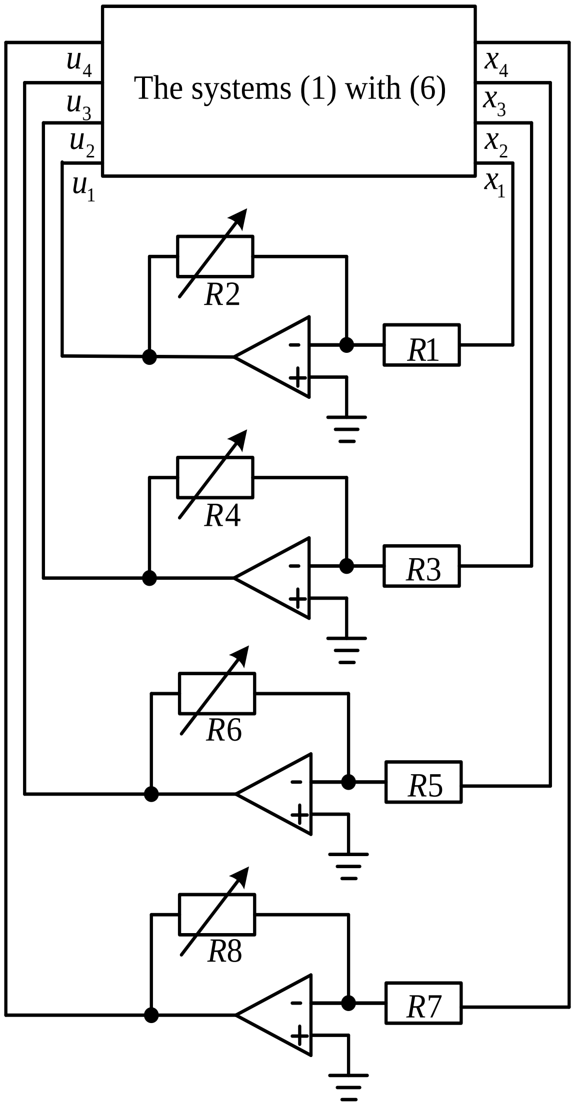

Figure 4: The hardware implementation diagram of the controller of (7), where $R1 = R3 = R5 = R7 = 1\,k\Omega$, $R2 = R4 = 9.25\,k\Omega$, and $R6 = R8 = 10.25\,k\Omega$.