COMPARISON OF RIMAN, LEBEG, LEBEG-STILTES, RIMAN-STILTES INTEGRALS

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ANNOTATION

This in the article Compare Riemann, Lebesgue, Lebesgue-Stiltes, Riemann-Stiltes integrals basics given .

Keywords: Faith integral , segment , integrable function , theorem , Lebesgue integral .

1. Comparison of Riemann and Lebesgue integrals.

First of all, let's start with the definitions of Riemann and Lebesgue integrals and information about their existence:

For example integration. If a function $[a,b]$ defined on a segment $f(x)$ is given, then according to Riemann's idea, $[a,b]$ the segment would be divided into segments $\Delta_1, \Delta_2, ..., \Delta_n$ with lengths n , and by choosing an arbitrary point from each segment ζ_k, the integral sum would be as follows

$$S_n = \sum_{k=1}^{n} f(\zeta_k) \cdot \Delta_k$$

were salted. If $\max \Delta_k \to 0$ there $(n \to \infty)$ is a limit of the sequence and this limit $[a,b]$ does not depend on $f(x)$ the method of dividing the segment into segments and the selection of points from each segment , then this limit ζ_k is called the segment-wise Riemann integral $\int_a^b f(x)dx$ of the function and $[a,b]$ is defined as was

Lebesgue's integral: A defines the exact lower and exact upper bounds of the function $[a,b]$ defined $f(x)$ in the dimensional set on the segment E with $[a,b]$ and , and divides B the segment n into parts as follows:

$$A = y_0 < y_1 < y_2 < ... < y_{n-1} < y_n = B$$

Then $l_k (k = \overline{1,n})$ with

$$e_k = \{x \in E : y_{k-1}\} \leq f(x) < y_k, \quad k = \overline{1,n}\}$$

sets are defined. In this case, $f(x)$ since the function is dimensional, $e_k = (k = \overline{1,n})$ the sets are dimensional.

This

$$S = \sum_{k=1}^{n} y_{k-1} \cdot \mu(e_k); \qquad S = \sum_{k=1}^{n} y_k \cdot \mu(e_k)$$

totals are made.

If $\alpha_n (\alpha_n = \max_{1<k<n} [y_k - y_{k-1}])$ $\qquad y_k - y_{k-1}$

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When tending to zero $(n \to \infty)$ s and S if there is a limit of sums equal to each other and this limit y_k does not depend on the selection of points, then this limit is called the Lebesgue integral $f(x)$ of the function E on the set and $(L)\int_E f(x)dx$ is written in the formBefore giving the Riemann and Lebesgue theorems, let us give the following definition. Definition 1. If the dimension of the set of discontinuities of a E given $f(x)$ function in a dimensional set is zero, then $f(x)$ the function E is called an almost continuous function in the set.

Now let's state the theorems about the existence of Riemann and Lebesgue integrals without proof:

Theorem 1. (On the existence of the Riemann integral). $[a,b]$ for a Riemannian integral of a function to exist in a segment , it is necessary and sufficient that it is bounded and almost continuous in this segment. $f(x)$

Theorem 2. On the existence of the Lebesgue integral. If $f(x)$ a function E is dimensional and bounded on a dimensional set, then it has a Lebesgue integral.

Theorem 3. If $[a,b]$ a segment $f(x)$ has a Riemann integral for a function, then there is also a Lebesgue integral for this function, and these integrals are mutually equivalent.

It can be seen from the above theorems that the concept of Riemann integral cannot be applied to many important functions used in mathematics.

For example, if we take the Dirichlet function as a function $[a,b]$ defined on a segment $f(x)$, i.e

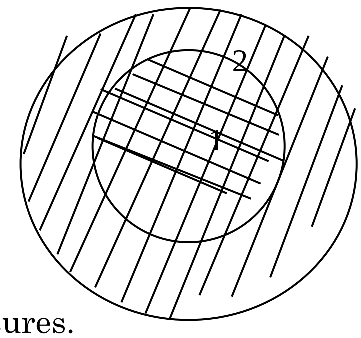
$$f(x) = \begin{cases} 1, & agar \quad x - ratsional \;\; son \; bo'lsa, \\ 0, & agar \quad x - irratsional \;\; son \; bo'lsa, \end{cases}$$

then the Riemann integral of this function does not exist. But the Lebesgue integral of this function exists.

Based on the information given above, the sets of integrable functions in the sense of Riman and Lebesgue can be described in the form of an Euler-Venn diagram as follows:

1-Integrable in the Riemannian sense set of functions.

2-Integrable in the sense of Lebesgue set of functions.



2. Lebesgue and Lebesgue-Stiltes comparing integrals.

First, let's talk about Stiltes and Lebesgue-Stiltes measures.

It is known $[a,b]$ that the Lebesgue measure of a segment is the length of this segment $b-a$. However, $[a,b]$ a segment and its subsets can be measured in a different, more general way.

Suppose we are given a $[a,b]$ left-continuous and monotonically non-decreasing function defined on a segment $F(x)$. Using this function, let's determine the dimensions $[a,b]$ of the segment, $[a,b)$ and $(a,b]$ the half-intervals, and (a,b) the interval, respectively, as follows:

$$\left.\begin{aligned} m[a,b] &= F(b+o) - F(a) \\ m[a,b) &= F(b) - F(a) \\ m(a,b] &= F(b+o) - F(a+o) \\ m(a,b) &= F(b) - F(a+o) \end{aligned}\right\} \qquad (1)$$

Let us denote by $[a,b]$ the system consisting of half-intervals of H all forms of the segment $[\alpha,\beta]$. H It is clear that the system is half a nation. According to (1) for any $[\alpha,\beta) = \in H$

$$m[\alpha,\beta) = F(\beta) - F(\alpha) \qquad (2)$$

Divides with equality.

Definition 2. If $F(x)$ the function $[a,b]$ is a left-continuous and monotonically non-decreasing function defined on a segment, and H the system is a system of half-intervals in $[a,b]$ all $[\alpha,\beta)$ forms of the segment, then m the set function H defined by equality (2) in the system F is the Stiltes measure generated by the function is called $F(x)$ The function is called a style function.

By continuing G this measurement to H the minimal loop containing the system , it is divided into $Z(H)$- additive μ_F measure. This measure is called the Lebesgue–Stiltes measure $F(x)$ corresponding to (or $F(x)$ generated by) the function. $F(x)$ and the function μ_F is called the function that generates the measure.

Definition 3. $[a,b]$ The Lebesgue integral obtained by the Lebesgue-Stiltes measure of the function μ_F defined in the segment is called the Lebesgue-Stiltes integral and is defined as follows. $f(x)$

$$\int_a^b f(x)dF(x)$$

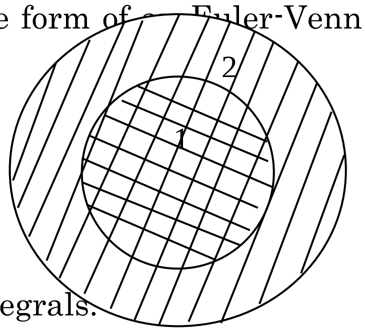
The following theorem can also be given.

Theorem 1. If $F(x)$ is an absolutely continuous function, then

$$\int_a^b f(x)dF(x) = \int_a^b f(x) \cdot F`(x)dx$$

Equality is appropriate. So, in this case, the Lebesgue-Stiltes integral becomes the Lebesgue integral.

Then, based on definitions 2, 3 and Theorem 4, the set of integrable functions in the sense of Lebesgue and Lebesgue-Stiltes can be described in the form of a Euler-Venn diagram as follows:



1 in the Lebesgue sense
a set of integrable functions.

2- Lebesgue – in the sense of Stiltes
a set of integrable functions.

3. Comparison of Riemann and Riemann-Stiltes integrals.

Let us give information about the Riemann-Stiltes integral.

$[a,b]$ and $\varphi(x)$ function $[a,b]$ defined in the segment be given. we divide $f(x)$ the segment $a_0, a_1, ..., a_n$ arbitrarily n into pieces with points:

$$a = a_0 < a_1 < ... < a_{n-1} < a_n = b$$

$$\alpha_n = \max_{1<k<n}(a_k - a_{k-1}).$$

each $[a_{k-1}, a_k]$ segment x_k, this

$$S_n = \sum_{k=1}^{n} f(x_k) \times (\varphi(a_k) - \varphi(a_{k-1}))$$

we make the sum.

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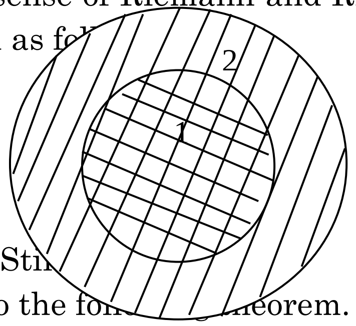
Definition 4. If, α_n when tending to zero , S_n the sum $[a,b]$ tends to a specific limit regardless of $f(x)$ how the segment is divided and how the points are chosen, then the value of this limit x_k is called the Riemann-Stiltes integral of the function obtained by the function on the $[a,b]$ segment and is written as follows: $\varphi(x)$

$$\lim_{\alpha_n \to \infty} S_n = \int_a^b f(x) > d\varphi(x).$$

It is called $f(x)$ an integrable function, $\varphi(x)$ and an integrating function.

It is clear from this definition that $\varphi(x) = x$ the Riemann integral derives from the Riemann-Stiltes integral if taken in a special case.

herefore, the sets of integrable functions in the sense of Riemann and Riemann–Stiltes can be described in the form of an Euler–Venn diagram as follows:



1. In the Riemann sense a set of integrable functions.

2. Rieman – in the sense of Stiltes a set of integrable functions.

4. Comparison of Riemann-Stiltes and Lebesgue-Stiltes integrals

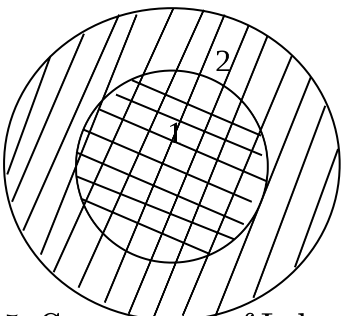
To make this comparison, we restrict ourselves to the following theorem.

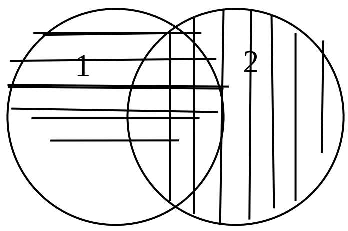
Theorem 5. If $[a,b]$ the segment contains $f(x)$ continuous and $\varphi(x)$ finite-variable functions, then $f(x)$ the function $\varphi(x)$ has a Riemann–Stiltes integral over the function and is equal to the Lebesgue–Stiltes integral of this function.

Here, based on the above information, the set of integrable functions in the sense of Riemann-Stiltes and Lebesgue-Stiltes can be described in the form of Euler-Venn diagram as follows:

1. Riemann is a set of integrable functions in the sense of Stiltes .

2. Lebesgue is a set of integrable functions in the sense of Stiltes.





5. Comparison of Lebesgue and Riemann-Stiltes integrals.

REFERENCES

1.SH.A.Ayupov, MMIbragimov, KKKhudoyberganov. "Examples and problems from functional analysis". Nukum "Knowledge". 2009

2. TASarimsakov. "Theory of functions of a real variable". Tashkent. "Teacher". 1982

3. T. Azlarov, H. Mansurov. "Mathematical Analysis". Part 1. Tashkent. "Teacher". 1994