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QUATERNION NUMBERS TO ' PLATE SOLVING THE QUADRATIC EQUATION

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ABSTRACT

The quadratic equation is solved using the operations defined in the set of quarternion numbers. The quadratic equation was solved in a special case, and its roots were found. The theorem on the solution of the quadratic equation is given, the formula for finding the square root of the number of quarternions is given.

Keywords: imaginary unit, guiding vectors, quaternion numbers, distribution property, biquadratic equation, infinitely many, square root.

INTRODUCTION

Usually, the multiplication of quaternions is determined using a multiplication table of abstract units and scalar units, and then a formula is derived using the distribution property. When multiplying quaternions, the property of associativity, distributivity with respect to additives is fulfilled, but in general it does not have the property of commutativity. However, if the parts of the vector are collinear, the commutativity property holds when multiplying quaternions. Since the commutativity property does not hold in the general case, the short multiplication formulas are incorrect, and on their basis formulas for solving a quadratic equation over a field of real and complex numbers are derived.

LITERATURE ANALYSIS AND METHODOLOGY

The structure of quaternions, operations on them [1] functions that reflect real numbers into quaternion numbers using the rules of differentiation and Füter's property have been studied [2] rotational movement of quaternions, operations on them, the movement of their three-dimensional vector part, the rules of rotation have been studied [3,4]. Analysis, induction methods were used in this article.

RESULTS

Quaternion numbers are a very convenient and effective method for solving problems with circular motion [1] . Quaternion numbers are a generalization of complex numbers, and if a complex number $i^2=-1$has one abstract unit satisfying the condition , quaternion numbers have three such abstract units .

$$\lambda=\lambda_0+\lambda_1 i+\lambda_2 j+\lambda_3 k \quad (1)$$

is defined in the form of Here, $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ s are given real numbers i, j, k, and s are defined by equations as abstract units that $i^2 = j^2 = k^2 = -1$ can be viewed as direction vectors in three - dimensional coordinate spaces .

The following

$$\vec{\lambda} = \lambda_1 \vec{i} + \lambda_2 \vec{j} + \lambda_3 \vec{k}$$

vector is the abstract or vector part of a given quaternion number $\vec{v}(\lambda)$ such as let's define . λ_0 while given quaternion of the thigh real part like bo ' lib $\operatorname{Re}\lambda$ designation if we enter , then given quaternion thigh

$$\lambda = \operatorname{Re}\lambda + \vec{v}(\lambda) \quad (2)$$

can be expressed as

Given $\lambda = \operatorname{Re}\lambda + \vec{v}(\lambda)$ and $\mu = \operatorname{Re}\mu + \vec{v}(\mu)$ product of quaternion numbers [1,2]

$$\lambda \cdot \mu = \operatorname{Re}\lambda \cdot \operatorname{Re}\mu - (\vec{v}(\lambda), \vec{v}(\mu)) + \operatorname{Re}\lambda \cdot \vec{v}(\mu) + \operatorname{Re}\mu \cdot \vec{v}(\lambda) + \vec{v}(\lambda) \times \vec{v}(\mu) \quad (3)$$

determined by the formula, where $(\vec{v}(\lambda), \vec{v}(\mu))$ - $\vec{v}(\lambda)$ and is $\vec{v}(\mu)$ the scalar product of vectors , and $\vec{v}(\lambda) \times \vec{v}(\mu)$ - $\vec{v}(\lambda)$ and $\vec{v}(\mu)$ represents the vector product of vectors .

Usually, the multiplication of quaternions is determined using the table of multiplication of abstract units and scalar units, and then formula (3) is derived using the distributive property. When multiplying quaternions , the property of associativity and distributivity is fulfilled with respect to the addends , but in general it does not have the property of commutativity. However, if the vector parts are collinear, then the commutativity property is satisfied when multiplying quaternions . In general, the short multiplication formulas are incorrect due to the non-fulfillment of the commutativity property , and based on them , the formulas for solving the quadratic equation over the field of real and complex numbers are derived.

learning how to solve a quadratic equation .

First for convenience

$$x^2 = c \quad (4)$$

the equation let's see Here _ $c = c_0 + c_1 i + c_2 j + c_3 k$ - given quaternion number, $x = x_0 + x_1 i + x_2 j + x_3 k$ while looking for for now unknown quaternion number.

to the square lifting _ _ two one different quaternion thigh multiple in the form of looking after if , then from formula (3). using equation (4).

$$x_0^2 - |\vec{v}(x)|^2 + 2x_0 \vec{v}(x) = c_0 + \vec{v}(c)$$

in the form of our writing possible will be _ _ This is equality

$$\begin{cases} x_0^2 - |\vec{v}(x)|^2 = c_0 \\ 2x_0 \vec{v}(x) = \vec{v}(c) \end{cases} \quad (5)$$

equalities only when done o ' rinli will be _ _

Let's assume $\vec{v}(c) \neq 0$. In that case $x_0 \neq 0$ from being $\vec{v}(x) = \dfrac{\vec{v}(c)}{2x_0}$ being

$$x_0^2-\frac{\left|\vec{v}(x)\right|^2}{4x_0^2}=c_0$$

we will have equality . It x_0 was produced to relatively biquadratic from Eq

$$\left(x_0^2\right)_{1,2}=\frac{c_0\pm\sqrt{c_0^2+\left|\vec{v}(x)\right|^2}}{4x_0^2}=\frac{c_0\pm\sqrt{|c|^2}}{2}=\frac{c_0\pm|c|}{2}$$

to have we will be _ From this while $x_0^2>0$ that in consideration will receive if , then 2 roots of equation (4) arise .

$$x=\pm\sqrt{\frac{c_0+|c|}{2}},\vec{v}(x)=\frac{\vec{v}(c)}{2x_0} \tag{6}$$

or

$$x=\pm\left(\sqrt{\frac{c_0+\sqrt{c_0^2+c_1^2+c_2^2+c_3^2}}{2}}+\frac{c_1i+c_2j+c_3k}{\sqrt{2\left(c_0+\sqrt{c_0^2+c_1^2+c_2^2+c_3^2}\right)}}\right) \tag{7}$$

Let's say $\vec{v}(c)=0$. Then (5) Eqs in the system $x_0=0$ or $\vec{v}(x)=0$ the fact that come comes out If $\vec{v}(x)=0$ if _ _ $x_0^2=c$ that is , that is $c_0\geq 0$ just in case o ' rinli being _$x=x_0=\pm\sqrt{c_0}=\pm\sqrt{c}$ in this $\vec{v}(c)=0$ will be If$c_0<0$ if , then _ _ $\vec{v}(x)\neq 0$ be , , $x_0=0$_ _$\left|\vec{v}(x)\right|=\sqrt{-c_0}$ ie three zero - dimensional in space of the sphere all points expressing , Eq infinite many p _ solution have will be _ _

So by doing the following theorem o ' rinli will be _ _

Theorem . In the form of (4). the equation is always to the solution have become _ _ _

1) $c=0$ at $x=0$ two multiple to the root ;
2) $\vec{v}(c)\neq 0$ in (6) or (7) by formula to 2 defined roots ;
3) $\vec{v}(c)=0,\ c_0>0$ at $x=\pm\sqrt{c_0}$ with to be determined by 2 roots;
4) $\vec{v}(c)=0$, $c_0<0$ when _ _ $x_1^2+x_2^2+x_3^2=-c_0,c_0<0$, condition satisfactory three zero - dimensional in space of the sphere all points expressing , Eq infinite many p _ to the solution have will be _ _

DISCUSSION

Multiplication of quaternions is defined using a table of multiplication of abstract units and scalar units, and then formulated using the distributive property. When multiplying quaternions , the property of associativity and distributivity is fulfilled with respect to the addends , but in general it does not have the property of commutativity. However, if the vector parts are collinear, then the commutativity property is satisfied when multiplying quaternions . In general, because the property of commutativity is not fulfilled, short multiplication formulas are wrong , that is , if we want to square a quaternion number, the quaternion number consists of one real part and three abstract parts, taking into account the real part If we consider the first abstract part and the second and third parts as separate numbers, we get the result as the square of the sum, but if we consider the real part and the second abstract part as separate numbers as the square of the sum, the result is always equal to the result in the first case In the third case, we get the same result, based on which the formulas for solving the quadratic equation over the field of real and complex numbers are derived.

CONCLUSION

Quaternion number square root

$$\sqrt{c}=\begin{cases}0, & agar\ \ c=0\\ \pm\left(\sqrt{\dfrac{c_0+|c|}{2}}+\dfrac{\vec{v}(c)}{\sqrt{2(c_0+|c|)}}\right), & agar\ \ \vec{v}(c)\neq 0\\ \pm\sqrt{c_0}, & agar\ \ \vec{v}(c)=0, c_0>0\\ x_1 i+x_2 j+x_3 k, (x_1,x_2,x_3): x_1^2+x_2^2+x_3^2=-c_0, & agar\ \ \vec{v}(c)=0, c_0<0\end{cases}$$

through the formula is determined .

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