

APPLICATIONS OF THE DERIVATIVE

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ANNOTATION

This article talks about determining the conditions for the function to remain constant in some interval, whether it is increasing or decreasing, whether it is convex or concave, turning points using the derivative, necessary and sufficient conditions. In each section, assertions are explained and reinforced with examples.

Key words: increasing, decreasing, differentiable, constant, interval, intercept, convex, concave, turning point, positive, negative

1. Condition of invariance of the function

Let the function $f(x)$ be differentiable in the interval (a, b) . In order for the function $f(x)$ not to change in the interval (a, b) , i.e. for $f(x)=C$, it is necessary and sufficient that its derivative at all points of this interval is equal to zero.

Proof. The need is obvious. Because if the function is constant, $f'(x)=0$ at all points.

Sufficiency. According to the condition, the function $f(x)$ is differentiable in the interval (a, b) , that is, there is a finite derivative $f'(x)$ for $\forall x \in (a, b)$ and $f'(x)=0$. Now let's take the points $\forall x_1, x_2 \in (a; b)$ where $x_1 < x_2$. The considered function $f(x)$ satisfies all the conditions of Lagrange's theorem in the section $[x_1; x_2]$. So, such a point c belonging to the interval $(x_1; x_2)$ is found,

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1) \quad (1)$$

equality will be appropriate. According to the condition of the theorem, $f'(x)=0$ for $\forall x \in (a; b)$, from which $f'(c)=0$, and it follows from equality (1) that $f(x_2)-f(x_1)=0$.

Thus, the values of the function $f(x)$ at any two points of the interval $(a; b)$ are mutually equal. So, the function will be invariant.

This leads to the following result, which plays an important role in integral calculus.

The result. If the functions $f(x)$ and $g(x)$ have finite derivatives $f'(x)$ and $g'(x)$ in (a, b) , and in this interval $f'(x) = g'(x)$ the equality holds, then the functions $f(x)$ and $g(x)$ differ from each other by a constant number:

$$f(x) = g(x) + C, \quad C = \text{const.}$$

Indeed, by condition $(f(x)-g(x))' = C' = 0$. Based on theorem 1, it follows that $f(x)-g(x)=C$, that is, $f(x)=g(x)+C$ is valid.

2. Monotonicity condition of the function in the set and at the point. Here we show that it is possible to determine the monotonicity of a function using the derivative of a function.

Let the function $f(x)$ be differentiable on the interval (a, b) . If $f'(x) > 0$ for $\forall x \in (a, b)$, then (a, b) is increasing on the interval I .

Proof. Let $x_1, x_2 \in (a; b)$ and $x_1 < x_2$. Clearly, the function $f(x)$ in the section $[x_1; x_2]$ satisfies all conditions of Lagrange's theorem. According to this theorem, there exists $c \in (x_1; x_2)$ such that $f(x_2)-f(x_1)=f'(c)(x_2-x_1)$

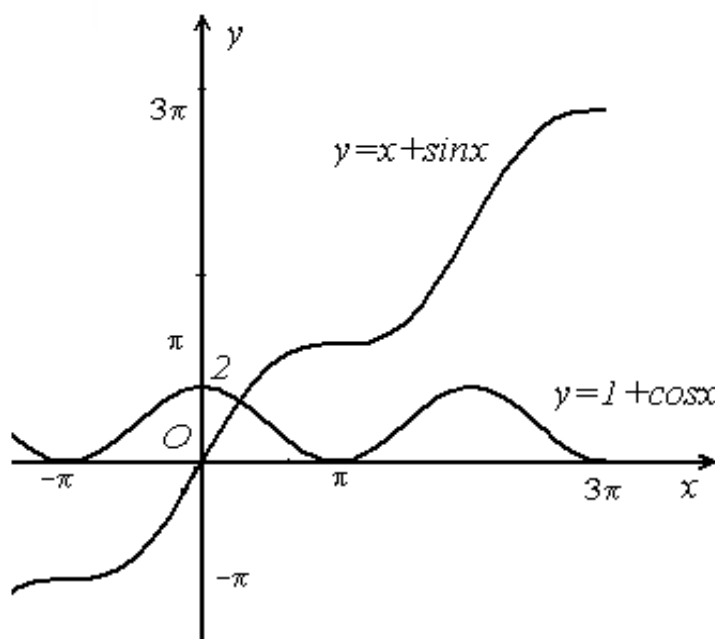
equality will be appropriate. From this equality and $f'(c) > 0$, it follows that $f(x_2) > f(x_1)$.

This means that the function $f(x)$ is exponentially increasing. This is a sufficient condition.

This function $y = x^3$ is strictly increasing in the interval $(-r, r)$ $r \in \mathbb{R}$, but its the derivative is equal to zero at the point $x = 0$.

These examples show that the conditions of the above theorem are the only sufficient condition for the function to be strictly increasing (decreasing).

A similar function $f(x) = x + \sin x$ is also strictly increasing in the definition domain, but its derivative $f'(x) = 1 + \cos x$ at infinitely many points ($x = \pi + 2\pi n, n \in \mathbb{Z}$) will be zero.



Example 1. Find the monotone intervals of this function $f(x) = x - \ln x$.

Solving. The function is defined in the interval $(0; +\infty)$. Its derivative is $f'(x) = 1 - 1/x$. According to the above sufficient condition, if $1 - 1/x > 0$, i.e. if $x > 1$, is increasing; if $1 - 1/x < 0$, that is, if $x < 1$, the function is decreasing. Thus, the function is decreasing in the interval $(0; 1)$ and increasing in the interval $(1; +\infty)$.

3. Monotonicity condition of the function at the point. So far, we have introduced and studied the concepts of increasing and decreasing functions with respect to an interval. In some cases, it is useful to look at these concepts in relation to the point.

Suppose that the function $f(x)$ is defined in the interval (a, b) and let $x_0 \in (a; b)$.

Description. If such a neighborhood of the point x_0 ($x_0 - \delta; x_0 + \delta$) is found, $f(x) < f(x_0)$ ($f(x) > f(x_0)$) when $x < x_0$, and f when $x > x_0$ If $f(x) > f(x_0)$ ($f(x) < f(x_0)$), then the function $f(x)$ is called increasing (decreasing) at the point x_0 .

Let the function $f(x)$ be differentiable at the point $x_0 \in (a; b)$. If $f'(x_0) > 0$ ($f'(x_0) < 0$), then the function $f(x)$ is increasing (decreasing) at this point.

At the points where the derivative of the function becomes zero, the function can increase or decrease. For example, the derivative of the function $y = x^5$ is zero at the point $x = 0$, but the function is increasing at this point; The derivative of the function $y = -x^5$ is also zero at $x = 0$, but it is not difficult to see that this function is decreasing at $x = 0$.

But a function that is increasing at a point x_0 does not necessarily have to be increasing around this point.

Let this $f(x) = \begin{cases} x + x^2 \sin \frac{2}{x}, & \text{agar } x \neq 0, \\ 0, & \text{agar } x = 0 \end{cases}$ function is given. This function has a derivative at

all points. Indeed, for $x \neq 0$ $f'(x) = 1 + 2x \sin \frac{2}{x} - 2 \cos \frac{2}{x}$, for $x = 0$ $f'(0) = 1 > 0$. Therefore, the

given function is increasing at the point $x = 0$.

Since the function $f(x)$ itself is increasing at the point $x = 0$, it has a derivative around this point $\forall (-\delta; \delta)$, but it is not monotonic around this point.

If there is a derivative of the function $f(x)$ at the point x_0 , continuous and $f'(x_0) > 0$, then there is a neighborhood of the point x_0 such that $(x_0 - \delta; x_0 + \delta)$ where the function $f(x)$ will be watery.

4. Convexity and concavity of the curve. Let's say that the function $f(x)$ has a derivative $f'(x_0)$ at the point $x = x_0$, that is, it is possible to perform a non-vertical test from the point $M(x_0, f(x_0))$ of the graph of the function.

Description. If there is such a neighborhood of the point $x = x_0$, and the segment of the curve $y = f(x)$ corresponding to the points in this neighborhood is below (above) is located, then the function $f(x)$ is called convex (concave) at the point $x = x_0$.

If a curve is convex (concave) at all points of an interval, then this line is called convex (concave) in that interval.

If in some interval $f''(x) > 0$ ($f''(x) < 0$), then the curve $y = f(x)$ is concave (convex) in this interval.

An example. Determine the intervals of concavity and convexity of the graph of this function $y = x^5$.

Solving. We find the second derivative of the function: $y'' = 20x^3$. Hence, if $x > 0$ then $y'' > 0$, if $x < 0$ then $y'' < 0$. So, the curve is convex in the interval $(-\infty; 0)$, and concave in the interval $(0; +\infty)$.

5. Inflection point of the curve. Now we introduce the concept of inflection point of a curve.

Definition 2. If such a neighborhood of the point x_0 ($x_0 - \delta; x_0 + \delta$) is found, and the function $f(x)$ is concave (convex) in the interval $(x_0 - \delta; x_0)$, and convex (concave) in the interval $(x_0; x_0 + \delta)$, then In this case, the point x_0 is called the inflection point of the curve $y = f(x)$.

If there is an attempt at a turning point, it crosses the curve.

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