ON ONE BOUNDARY PROBLEM FOR A PARABOLIC-HYPERBOLIC EQUATION OF THE THIRD ORDER, WHEN CHARACTERISTICS OF THE FIRST ORDER OPERATOR PARALLEL TO THE X-AXIS

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ANNOTATION

Bu ishda account type ўzgarish chizigiga ega bulgan beshburchakli sohada $\left(a\frac{\partial}{\partial x}+c\right)(Lu)=0$ kurinishdaghi uchinchi tartibli parabolic-hyperbolic tenglama uchun bitta chegaraviy masala қўyiladi va tadқiқ etiladi.

Keywords ; unknown functions, complication, theorem, equation, open segment, point, unknown function, triangle .

INTRODUCTION

Per elapsed time studies on boundary value problems for equations of the third and higher orders of parabolic-hyperbolic type developed in a broad sense , and is currently expanding in directions of complication of equations and the area of their consideration, as well as generalizations of the problems of equations considered for them (for example, see [1] - [7] and others).

FORMULATION OF THE PROBLEM

In this article , in the area of G \u200b\ u200bthe plane xOy the parabolic-hyperbolic equation is considered e third order of the form

$$\left(a\frac{\partial}{\partial x}+c\right)(Lu)=0, \qquad \text{(one)}$$

where $a,c\in R$, and $a\neq 0$; $G=G_1\cup G_2\cup G_3\cup G_4\cup J_1\cup J_2\cup J_3$, and a G_1- rectangle with vertices at points $A(0,0)$, $B(1,0)$, $B_0(1,1)$, $A_0(0,1)$; G_2- triangle with vertices at points A, B, $C(1/2,-1/2)$; G_3- triangle with vertices at points A . $D(-1/2,1/2)$. A_0 ; G_4- triangle with vertices at points B, B_0, $E(3/2,1/2)$; J_1- open segment with vertices at points A, B; J_2- open segment with vertices at points A, A_0; J_3-open segment with vertices at points B, B_0; $u=u(x,y)-$unknown function and

$$Lu=\begin{cases}u_{xx}-u_y, & (x,y)\in G_1,\\ u_{xx}-u_{yy}, & (x,y)\in G_j,\ j=2,3,4.\end{cases}$$

For equation (1) in the domain G , the following problem can be formulated and investigated :

A task M_{ac}. It is required to find a function $u(x,y)$ that is 1) continuous in $\overline{G}$ and $G\setminus J_1\setminus J_2\setminus J_3$ has in the domain continuous derivatives involved in equation (1), u_x and u_y are continuous in G up to the part of the boundary of the region G specified in the boundary conditions ; 2) satisfies equation (1) in the domain $G\setminus J_1\setminus J_2\setminus J_3$; 3) satisfies the following boundary conditions:

$$u|_{BC}=\psi_1(x),\ 1/2\le x\le 1; \quad (2)$$

$$u|_{AD}=\psi_2(x),\ -1/2\le x\le 0; \quad (3)$$

$$u|_{B_0E}=\psi_3(x),\ 1\le x\le 3/2; \quad (4)$$

$$\left.\frac{\partial u}{\partial n}\right|_{AC}=\psi_4(x),\ 0\le x\le 1/2; \quad (5)$$

$$\left.\frac{\partial u}{\partial n}\right|_{AD}=\psi_5(x),\ -1/2\le x\le 0; \quad (6)$$

$$\left.\frac{\partial u}{\partial n}\right|_{A_0D}=\psi_6(x),\ -1/2\le x\le 0; \quad (7)$$

$$\left.\frac{\partial u}{\partial n}\right|_{BE}=\psi_7(x),\ 1\le x\le 3/2; \quad (8)$$

$$\left.\frac{\partial u}{\partial n}\right|_{B_0E}=\psi_8(x),\ 1\le x\le 3/2; \quad (9)$$

and 4) the following continuous bonding conditions:

$$u(x,+0)=u(x,-0)=\tau_1(x),\ 0\le x\le 1; \quad \text{(ten)}$$

$$u_y(x,+0)=u_y(x,-0)=\nu_1(x),\ 0\le x\le 1; \quad \text{(eleven)}$$

$$u(+0,y)=u(-0,y)=\tau_2(y),\ 0\le y\le 1; \quad (12)$$

$$u_x(+0,y)=u_x(-0,y)=\nu_2(y),\ 0\le y\le 1; \quad (13)$$

$$u_{xx}(+0,y)=u_{xx}(-0,y)=\mu_2(y),\ 0<y<1; \quad \text{(fourteen)}$$

$$u(1+0,y)=u(1-0,y)=\tau_3(y),\ 0\le y\le 1; \quad \text{(fifteen)}$$

$$u_x(1+0,y)=u_x(1-0,y)=\nu_3(y),\ 0\le y\le 1. \quad (16)$$

Here $\psi_j\left(j=\overline{1,8}\right)$ – given sufficiently smooth functions and, $\tau_i, \nu_i\ \left(i=\overline{1,3}\right)$, μ_2 – unknown yet sufficiently smooth functions to be determined ; n – internal normal to line $x+y=-1\ (AD)$ or $x-y=-1\ (A_0D)$ or $x-y=1\ (BE)$ or $x+y=2\ (B_0E)$.

PROBLEM RESEARCH

Theorem. If $\psi_1(x)\in C^3[1/2,1]$, $\psi_2(x)\in C^3[-1/2,0]$, $\psi_3(x)\in C^3[1,3/2]$, $\psi_4(x)\in C^2[0,1/2]$, $\psi_5(x),\psi_6(x)\in C^2[-1/2,0]$, $\psi_7(x),\psi_8(x)\in C^2[1,3/2]$ and the matching conditions $\psi_4(0)=\psi_5(0)$, $\psi_5'(-1/2)=\psi_6'(-1/2)$, $\psi_7'(3/2)=\psi_8'(3/2)$, $\tau_1(0)=\tau_2(0)$, $\nu_1(0)=\tau_2'(0)$, $\tau_1(1)=\tau_3(0)$, are satisfied $\nu_1(1)=\tau_3'(0)$, then the task M_{ac} admits a unique solution.

Proof. We will prove the theorem by the method of constructing a solution. To do this, we rewrite equation (1) in the form

$$u_{1xx} - u_{1y} = \omega_1(y) e^{-\frac{c}{a}x}, \ (x,y) \in G_1, \qquad (17)$$

$$u_{jxx} - u_{jyy} = \omega_j(y) e^{-\frac{c}{a}x}, \ (x,y) \in G_j, \ \ j = \overline{2,3,4}, \qquad \text{(eighteen)}$$

where the notation is introduced $u(x,y) = u_j(x,y), (x,y) \in G_j \ (j=4)$, and $\omega_j(y), \ j = \overline{1,4} -$ unknown yet sufficiently smooth functions to be determined .

Let us first consider the problem in the domain G_2. Solution of equation (18) satisfying conditions (10), (11) , at $j=2$ can be represented in the form

$$u_2(x,y) = \frac{1}{2}\left[\tau_1(x+y) + \tau_1(x-y)\right] + \frac{1}{2}\int_{x-y}^{x+y} \nu_1(t)\,dt - \frac{1}{2}\int_0^y \omega_2(\eta)\,d\eta \int_{x-y+\eta}^{x+y-\eta} e^{-\frac{c}{a}\xi}\,d\xi . \ (19)$$

Substituting (19) into condition (5) and differentiating the resulting equality, we find

$$\omega_2(y) = \sqrt{2}\psi_4'(-y) e^{-\frac{c}{a}y}, \ -1/2 \le y \le 0 .$$

Further, substituting (19) into condition (2), after some simplifications, we obtain the first relation between the unknown functions $\tau_1(x)$ and $\nu_1(x)$ on the type change line J_1:

$$\tau_1'(x) + \nu_1(x) = \alpha_1(x), \ 0 \le x \le 1, \ (20)$$

where $\alpha_1(x) = \psi_1'\left(\frac{x+1}{2}\right) - \int_0^{(x-1)/2} \omega_2(\eta) e^{-\frac{c}{a}(x-\eta)}\,d\eta$.

Passing to equation (17) to the limit at $y \to 0$, we obtain the second relation between $\tau_1(x)$ and $\nu_1(x)$ on the line of change of type J_1:

$$\tau_1''(x) - \nu_1(x) = \omega_1(0) e^{-\frac{c}{a}x}, \ 0 \le x \le 1,$$

where $\omega_1(0) -$ is an unknown real number.

Eliminating the function from the last equation and from equation (20) $\nu_1(x)$ and integrating the resulting equation from 0 to x, we arrive at the equation

$$\tau_1'(x) + \tau_1(x) = \alpha_2(x) + \omega_1(0) h(x) + k_1, 0 \le x \le 1, \ (21)$$

where $\alpha_2(x) = \int_0^x \alpha_1(t)\,dt$ and is $h(x) = \int_0^x e^{-\frac{c}{a}t}\,dt = \begin{cases} \frac{a}{c}\left(1 - e^{-\frac{c}{a}x}\right), & c \ne 0, \\ x, \ c = 0, \end{cases}$ an $k_1 -$unknown constant.

When solving equation (21), there may be the following cases: 1°) $c \ne a$, $c \ne 0$; 2°) $c = a$; 3°) $c = 0$.

Consider the case 1°) $c \ne a$, $c \ne 0$. Solving equation (21) under the conditions

$$\tau_1(0) = \psi_2(0), \ \tau_1'(0) = \frac{1}{2}\psi_2'(0) + \frac{\sqrt{2}}{2}\psi_5(0), \ \tau_1(1) = \psi_1(1), \ (22)$$

we find the function unequivocally $\tau_1(x)$:

$$\tau_1(x)=\int_0^x e^{t-x}\alpha_2(t)\,dt+\omega_1(0)\frac{a}{c}\left[1-e^{-x}-\frac{a}{a-c}\left(e^{-\frac{c}{a}x}-e^{-x}\right)\right]+k_1\left(1-e^{-x}\right)+k_2e^{-x}, \quad (23)$$

where

$$k_2=\psi_2(0),\ \ k_1=\frac{1}{2}\psi_2'(0)+\frac{\sqrt{2}}{2}\psi_5(0)+\psi_2(0),$$

$$\omega_1(0)=\frac{c}{a}\left\{e-1-\frac{a}{a-c}\left(e^{\frac{a-c}{a}}-1\right)\right\}^{-1}\left\{\psi_1(1)e-\int_0^1 e^t\alpha_2(t)\,dt-k_1(e-1)-k_2\right\}.$$

Now consider case 2°) $c=a$. Then solving equation (21) under conditions (22), we have

$$\tau_1(x)=\int_0^x e^{t-x}\alpha_2(t)\,dt+\omega_1(0)\left(1-e^{-x}-xe^{-x}\right)+k_1\left(1-e^{-x}\right)+k_2e^{-x},$$

where

$$k_2=\psi_2(0).\ \ k_1=\frac{1}{2}\psi_2'(0)+\frac{\sqrt{2}}{2}\psi_5(0)+\psi_2(0).$$

$$\omega_1(0)=\frac{1}{e-2}\left[\psi_1(1)e-\int_0^1 e^t\alpha_2(t)\,dt-k_1(e-1)-k_2\right].$$

Finally, consider the case 3°) $c=0$. Solving equation (21) under conditions (22), we obtain

$$\tau_1(x)=\int_0^x e^{t-x}\alpha_2(t)\,dt+\omega_1(0)\left(x-1+e^{-x}\right)+k_1\left(1-e^{-x}\right)+k_2e^{-x},$$

where

$$k_2=\psi_2(0).\ \ k_1=\frac{1}{2}\psi_2'(0)+\frac{\sqrt{2}}{2}\psi_5(0)+\psi_2(0).$$

$$\omega_1(0)=\psi_1(1)e-\int_0^1 e^t\alpha_2(t)\,dt-k_1(e-1)-k_2.$$

Then the functions $\nu_1(x)$, $u_2(x,y)$ are found uniquely.

Now consider the problem in the area G_3. We write down the solution of equation (18) $(j=3)$ that satisfies the conditions (1 2), (13) :

$$u_3(x,y)=\frac{1}{2}\left[\tau_2(y+x)+\tau_2(y-x)\right]+\frac{1}{2}\int_{y-x}^{y+x}\nu_2(t)\,dt+\frac{1}{2}\int_0^x e^{-\frac{c}{a}\eta}d\eta\int_{y-x+\eta}^{y+x-\eta}\omega_3(\xi)\,d\xi. \quad (24)$$

Substituting (24) into conditions (6) and (7), after some calculations and transformations, we find

$$\omega_3(y)=\begin{cases}\sqrt{2}\psi_5'(-y)e^{-\frac{c}{a}y},\ 0\le y\le 1/2,\\ \sqrt{2}\psi_6'(y-1)e^{\frac{c}{a}(y-1)},\ 1/2\le y\le 1.\end{cases}$$

Further, substituting (24) into condition (3), after some calculations, we have the first relation between the unknown functions $\tau_2(y)$ and $\nu_2(y)$:

$$\nu_2(y)=\tau_2'(y)+\beta_1(y),\ 0\le y\le 1, \quad (25)$$

where $\beta_1(y)=\psi_2'\left(-\frac{y}{2}\right)-\int_0^{-\frac{y}{2}} e^{-\frac{c}{a}\eta}\omega_3(y+\eta)\,d\eta$.

Passing in equations (17) and (1 8) ($j=3$) to the limit at $x\to 0$, we have

$$\mu_2(y)-\tau_2'(y)=\omega_1(y),\ \mu_2(y)-\tau_2''(y)=\omega_3(y),\ y\in[0,1].$$

Eliminating the function from these equalities $\mu_2(y)$, we find

$$\omega_1(y)=\tau_2''(y)-\tau_2'(y)+\omega_3(y),\ y\in[0,1].\quad(26)$$

Next, move on to the area G_4. We write down the solution of equation (1 8) ($j=4$) that satisfies conditions (15), (16):

$$u_4(x,y)=\frac{1}{2}\left[\tau_3(y+x-1)+\tau_3(y-x+1)\right]+\frac{1}{2}\int_{y-x+1}^{y+x-1}\nu_3(t)\,dt+\frac{1}{2}\int_1^x e^{-\frac{c}{a}\eta}d\eta\int_{y-x+\eta}^{y+x-\eta}\omega_4(\xi)\,d\xi.\quad(27)$$

Substituting (27) into conditions (8) and (9), after some calculations and transformations, we find

$$\omega_4(y)=\begin{cases}-\sqrt{2}\psi_7'(y+1)e^{\frac{c}{a}(y+1)}, & 0\le y\le 1/2,\\ -\sqrt{2}\psi_8'(2-y)e^{\frac{c}{a}(2-y)}, & 1/2\le y\le 1.\end{cases}$$

Further, substituting (27) into condition (4), after some calculations, we have the first relation between the unknown functions $\tau_3(y)$ and $\nu_3(y)$:

$$\nu_3(y)=\tau_3'(y)+\beta_2(y),\ 0\le y\le 1,\quad(28)$$

where $\beta_2(y)=\psi_3'\left(\frac{3-y}{2}\right)-\int_1^{\frac{3-y}{2}} e^{-\frac{c}{a}\eta}\omega_4(y-1+\eta)\,d\eta$.

Now let's move on to the area G_1. Let us write the solution of equation (17) that satisfies conditions (10), (12), and (15):

$$u_1(x,y)=\int_0^y\tau_2(\eta)G_\xi(x,y;0,\eta)\,d\eta-\int_0^y\tau_3(\eta)G_\xi(x,y;1,\eta)\,d\eta+$$

$$+\int_0^1\tau_1(\xi)G(x,y;\xi,0)\,d\xi-\int_0^y\omega_1(\eta)\,d\eta\int_0^1 e^{-\frac{c}{a}\xi}G(x,y;\xi,\eta)\,d\xi,\quad(29)$$

where $G(x,y;\xi,\eta)$ is the Green's function of the 1-boundary value problem for the Fourier equation [10]:

$$G(x,y;\xi,\eta)=\frac{1}{2\sqrt{\pi(y-\eta)}}\sum_{n=-\infty}^{+\infty}\left\{\exp\left[-\frac{(x-\xi-2n)^2}{4(y-\eta)}\right]-\exp\left[-\frac{(x+\xi-2n)^2}{4(y-\eta)}\right]\right\},\ y>\eta.$$

Differentiating (29) with respect to x and tending x to zero and one, taking into account (25), (26) and (28), after lengthy calculations and transformations, we arrive at a system of Volterra integral equations of the second kind with respect to $\tau_2''(y)$ and $\tau_3'(y)$:

$$\tau_2''(y)+\int_0^y K_1(y,\eta)\tau_2''(\eta)\,d\eta+\int_0^y K_2(y,\eta)\tau_3'(\eta)\,d\eta=g_1(y),\quad(30)$$

$$\tau_3'(y)+\int_0^y K_3(y,\eta)\tau_3'(\eta)d\eta+\int_0^y K_4(y,\eta)\tau_2'(\eta)d\eta=g_2(y). \quad (31)$$

Solving system (30), (31), we find the functions $\tau_2''(y)$ and $\tau_3'(y)$, and thus the functions $\nu_2(y)$, $\nu_3(y)$, $\omega_1(y)$, $u_3(x,y)$, $u_4(x,y)$ and $u_1(x,y)$. Thus, we have found a solution to the problem in a unique way. The theorem has been proven.

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