STATEMENT AND INVESTIGATION OF ONE BOUNDARY PROBLEM FOR ONE PARABOLIC-HYPERBOLIC EQUATION OF THE THIRD ORDER IN A PENTAGONAL DOMAIN WITH THREE LINES OF TYPE CHANGE

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ANNOTATION

In the present work, we pose and study one boundary value problem for a parabolic- hyperbolic third-order equations of the form $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+c\right)(Lu)=0$ in a pentagonal region with three lines of change of type , one of the hyperbolic parts of which is a triangle, and the other two are rectangles.

Keywords ; parabolic-hyperbolic, open segment, equation, Volterran, integral equations.

INTRODUCTION

At present, the study of various boundary value problems for equations of the third and higher orders of the parabolic-hyperbolic type is being developed in a broad sense. (for example, see [1] - [7]).

In this article, we pose and study one boundary value problem for the equation

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+c\right)(Lu)=0, \qquad (1)$$

in the area of G the plane xOy, where $c\in R$, $G=G_1\cup G_2\cup G_3\cup G_4\cup J_1\cup J_2\cup J_3$, and a G_1 – rectangle with vertices at points $A(0;0)$, $B(1;0)$, $B_0(1,1)$, $A_0(0,1)$; G_2 – triangle with vertices at points $C(2,0)$. $E(1/2,-3/2)$. $D(-1,0)$; G_3 – rectangle with vertices at points A. D. $D_0(-1,1)$, A_0; G_4 – rectangle with vertices at points B, B_0, $C_0(2,1)$, $C(2,0)$; J_1 – open segment with vertices at points C, D; J_2 – open segment with vertices at points A, A_0; J_3 – open segment with vertices at points B, B_0, a

$$Lu=\begin{cases}u_{xx}-u_y, & (x,y)\in G_1,\\ u_{xx}-u_{yy}, & (x,y)\in G_j,\ j=2,3,4.\end{cases}$$

For equation (1), the following is set

A task M_{11c}. It is required to find a function $u(x,y)$ that is 1) continuous in $\overline{G}$ and $G\setminus J_1\setminus J_2\setminus J_3$ has in the domain continuous derivatives involved in the equation (2.1.1), moreover, u_x and u_y are continuous up to a part of the boundary of the region G, indicated in the boundary conditions ; 2) satisfies equation (2.1.1) in the domain $G\backslash J_1\backslash J_2\backslash J_3$; 3) satisfies the following boundary conditions:

$u(2,y)=\varphi_1(y),\ \ 0\le y\le 1;$ (2) $\quad u(-1,y)=\varphi_2(y),\ 0\le y\le 1;$ (3)

$u_x(-1,y)=\varphi_3(y),\ 0\le y\le 1;$ (four) $\quad u|_{EQ}=\psi_3(x),\ 1/2\le x\le 1;$ (5)

$u|_{CP}=\psi_2(x),\ 3/2\le x\le 2;$ (6) $\quad u|_{DE}=\psi_1(x),\ -1\le x\le 1/2;$ (7)

$\left.\dfrac{\partial u}{\partial n}\right|_{DE}=\psi_4(x),\ -1\le x\le 1/2;$ (eight)

and 4) satisfies the following bonding conditions:

$u(x,+0)=u(x,-0)=T(x),\ -1\le x\le 2;$ (9) $u_y(x,+0)=u_y(x,-0)=N(x),\ -1\le x\le 2;$ (ten)

$u_{yy}(x,+0)=u_{yy}(x,-0)=M(x),\ x\in(-1,0)\cup(0,1)\cup(1,2);$ (eleven)

$u(+0,y)=u(-0,y)=\tau_4(y),\ 0\le y\le 1;$ (12) $u_x(+0,y)=u_x(-0,y)=\nu_4(y),\ 0\le y\le 1;$ (13)

$u_{xx}(+0,y)=u_{xx}(-0,y)=\mu_4(y),\ 0<y<1;$ (fourteen) $u(1+0,y)=u(1-0,y)=\tau_5(y),\ 0\le y\le 1;$ (fifteen)

$u_x(1+0,y)=u_x(1-0,y)=\nu_5(y),\ 0\le y\le 1;$ (16) $u_{xx}(1+0,y)=u_{xx}(1-0,y)=\mu_5(y),\ 0<y<1.$ (17)

Here $\varphi_i\ \left(i=\overline{1,3}\right)$ and $\psi_j\left(j=\overline{1,4}\right)$ – given sufficiently smooth functions , n – the inner normal to the line $x-y=2\ \ (CE)$ or $x+y=-1\ \ (DE)$, a $Q(1,-1)$, $P(3/2,-1/2)$. Besides,

$$T(x)=\begin{cases}\tau_2(x),\ -1\le x\le 0,\\ \tau_1(x),\ 0\le x\le 1,\\ \tau_3(x),\ 1\le x\le 2;\end{cases}\quad N(x)=\begin{cases}\nu_2(x),\ -1\le x\le 0,\\ \nu_1(x),\ 0\le x\le 1,\\ \nu_3(x),\ 1\le x\le 2;\end{cases}\quad M(x)=\begin{cases}\mu_2(x),\ -1<x<0,\\ \mu_1(x),\ 0<x<1,\\ \mu_3(x),\ 1<x<2,\end{cases}$$

a $\tau_i,\ \nu_i,\ \mu_i\ \left(i=\overline{1,5}\right)$, - unknown yet sufficiently smooth functions.

The following theorem holds:

Theorem. If $\varphi_1(y)\in C^3[0,1]$, $\varphi_2(y)\in C^3[0,1]$, $\varphi_3(y)\in C^2[0,1]$, $\psi_1(x)\in C^3[-1,1/2]$, $\psi_2(x)\in C^3[3/2,2]$, $\psi_3(x)\in C^3[1/2,1]$, $\psi_4(x)\in C^2[-1,1/2]$ and the matching conditions $\tau_3(2)=\psi_2(2)=\varphi_1(0)$, $\tau_2(-1)=\psi_1(-1)=\varphi_2(0)$, $\psi_1(1/2)=\psi_3(1/2)$, $\tau_1(0)=\tau_4(0)=\tau_2(0)$, $\tau_1'(0)=\tau_2'(0)$, $\nu_1(0)=\tau_4'(0)$, $\tau_1(1)=\tau_5(0)=\tau_3(1)$, $\nu_1(1)=\tau_5'(0)$, are satisfied $\tau_1'(1)=\tau_3'(1)$, then the task M_{11c} admits a unique solution.

Proof. We will prove the theorem by the method of constructing a solution. To do this, we rewrite equation (1) in the form

$u_{1xx}-u_{1y}=\omega_1(x-y)e^{-cy},\ (x,y)\in G_1;$ (18)

$u_{ixx}-u_{iyy}=\omega_i(x-y)e^{-cy},\ (x,y)\in G_i,\ i=2,3,4,$ (19)

where the notation $u(x,y)=u_i(x,y),\ (x,y)\in G_i,\ i=\overline{1,4}$, and $\omega_i(x-y),\ i=\overline{1,4}$ - unknown yet sufficiently smooth functions.

First, consider equation (19) ($i=2$) in the region G_2. Its solution that satisfies the conditions (9), (10) is written as

$$u_2(x,y)=\frac{1}{2}\left[T(x+y)+T(x-y)\right]+\frac{1}{2}\int_{x-y}^{x+y}N(t)\,dt-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\omega_2(\xi-\eta)\,d\xi\,.\ \text{(twenty)}$$

Substituting (20) into condition (8) after some calculations and transformations, we find

$$\omega_2(x-y)=\sqrt{2}\psi_4'\left(\frac{x-y-1}{2}\right)e^{-\frac{c}{2}(x-y+1)},\ -1\le x-y\le 2\,.$$

Taking into account condition (7) from (20), after some calculations, we obtain the relation between the unknown functions $T(x)$ and $N(x)$:

$$T'(x)-N(x)=\alpha_1(x),\ -1\le x\le 2,\ 21)$$

where $\alpha_1(x)=\psi_1'\left(\frac{x-1}{2}\right)+\omega_2(x)\int_0^{-\frac{x+1}{2}}e^{-c\eta}d\eta\,.$

At $-1\le x\le 0$, equation (21) has the form

$$\tau_2'(x)-\nu_2(x)=\alpha_1(x),\ -1\le x\le 0\,.\ (22)$$

Substituting (20) into condition (5), we obtain the relation

$$\tau_2'(x)+\nu_2(x)=\delta_1(x),\ -1\le x\le 0,\ (23)$$

where

$$\delta_1(x)=\psi_3'\left(\frac{x+2}{2}\right)+\int_0^{\frac{x-2}{2}}e^{-c\eta}\omega_2(x-2\eta)\,d\eta\,.$$

From (22) and (23) we find

$$\tau_2'(x)=\frac{1}{2}\left[\alpha_1(x)+\delta_1(x)\right],\ \nu_2(x)=\frac{1}{2}\left[\delta_1(x)-\alpha_1(x)\right],\ -1\le x\le 0\,.\ (24)$$

Integrating the first of equalities (24) from -1 to x, we find

$$\tau_2(x)=\frac{1}{2}\int_{-1}^{x}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_1(-1),\ -1\le x\le 0\,.$$

At $1\le x\le 2$, equation (21) has the form

$$\tau_3'(x)-\nu_3(x)=\alpha_1(x),\ 1\le x\le 2\,.\ (25)$$

Substituting (20) into condition (6), we obtain the relation

$$\tau_3'(x)+\nu_3(x)=\delta_2(x),\ 1\le x\le 2,\ (26)$$

where

$$\delta_2(x)=\psi_2'\left(\frac{x+2}{2}\right)+\int_0^{\frac{x-2}{2}}e^{-c\eta}\omega_2(x-2\eta)\,d\eta\,.$$

From (25) and (26) we find

$$\tau_3'(x)=\frac{1}{2}\left[\alpha_1(x)+\delta_2(x)\right],\ \nu_3(x)=\frac{1}{2}\left[\delta_2(x)-\alpha_1(x)\right],\ 1\le x\le 2\,.\ (27)$$

Integrating the first of equalities (27) from 2 to x, we find

$$\tau_3(x)=\frac{1}{2}\int_2^{x}\left[\alpha_1(t)+\delta_2(t)\right]dt+\psi_2(2),\ 1\le x\le 2\,.$$

And for , $0\le x\le 1$equation (21) has the form

$\tau_1'(x)-\nu_1(x)=\alpha_1(x),\ 0\le x\le 1.$ (28)

Next, we G_1 rewrite equation (1) in the domain in the form

$u_{1xxx}-u_{1xy}+u_{1xxy}-u_{1yy}+cu_{1xx}-cu_{1y}=0.$

Passing in the last equation and in equation (19) $(i=2)$ to the limit at $y\to 0$, we obtain the second and third relations between the unknown functions $\tau_1(x)$, $\nu_1(x)$ and $\mu_1(x)$ on the type change line J_1:

$\tau_1'''(x)-\nu_1'(x)+\nu_1''(x)-\mu_1(x)+c\tau_1''(x)-c\nu_1(x)=0,\ 0\le x\le 1,$ (29)

$\mu_1(x)=\tau_1''(x)-\omega_2(x),\ 0\le x\le 1.$ (thirty)

Eliminating the functions and from equations (28), (29) and (30) $\nu_1(x)$ and $\mu_1(x)$ integrating the resulting equation from 0 to x, we arrive at the equation

$\tau_1''(x)-\left(1-\frac{c}{2}\right)\tau_1'(x)-\frac{c}{2}\tau_1(x)=\alpha_2(x)+k_1,\ 0\le x\le 1,$ (31)

where

$$\alpha_2(x)=\frac{1}{2}\alpha_1'(x)-\frac{1}{2}\alpha_1(x)-\frac{1}{2}\int_0^x\left[\omega_2(t)+c\alpha_1(t)\right]dt,$$

and k_1 –yet unknown constant.

When solving equation (31), there may be the following cases: 1°. $c\ne 0$, $c\ne -2$; 2°. $c=-2$; 3°. $c=0$.

Consider case 1°. In this case, solving equation (31) under the conditions

$$\tau_1(0)=\frac{1}{2}\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_3(-1).\ \ \tau_1'(0)=\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right].$$

$$\tau_1(1)=-\frac{1}{2}\int_1^2\left[\alpha_1(t)+\delta_2(t)\right]dt+\psi_2(2), \qquad (32)$$

find

$$\tau_1(x)=\frac{2}{2+c}\int_0^x\left[e^{x-t}-e^{\frac{c}{2}(t-x)}\right]\alpha_2(t)\,dt+\frac{2k_1}{2+c}\left[e^x-1-\frac{2}{c}\left(1-e^{-\frac{c}{2}x}\right)\right]+$$

$$k_2e^x+k_3e^{-\frac{c}{2}x},\ \ 0\le x\le 1,$$

where

$$k_3=\frac{1}{2+c}\left\{\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+2\psi_3(-1)-\left[\alpha_1(0)+\delta_1(0)\right]\right\},$$

$$k_2=\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right]+\frac{c}{2}k_3,\ k_1=\left[e-1-\frac{2}{c}\left(1-e^{-\frac{c}{2}}\right)\right]^{-1}\times$$

$$\times\left\{\frac{2+c}{2}\left[\psi_2(2)-k_2e-k_3e^{-\frac{c}{2}}\right]-\int_0^1\left[e^{1-t}-e^{\frac{c}{2}(t-1)}\right]\alpha_2(t)\,dt-\frac{c+2}{4}\int_1^2\left[\alpha_1(t)+\delta_2(t)\right]dt\right\}.$$

Consider case 2°. In this case, solving equation (31) under conditions (32), we find

$$\tau_1(x)=\int_0^x (x-t)e^{x-t}\alpha_2(t)dt+k_1\left[1+(x-1)e^x\right]+(k_2+k_3x)e^x,\ 0\le x\le 1,$$

where

$$k_2=\frac{1}{2}\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_3(-1).\ k_3=\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right]-k_2.$$

$$k_1=\psi_2(2)-(k_2+k_3)e-\int_0^1(1-t)e^{1-t}\alpha_2(t)dt-\frac{1}{2}\int_1^2\left[\alpha_1(t)+\delta_2(t)\right]dt.$$

Consider case 3°. In this case, solving equation (31) under conditions (32), we find

$$\tau_1(x)=\int_0^x e^{x-t}\alpha_3(t)dt+k_1\left(e^x-x-1\right)+k_2\left(e^x-1\right)+k_3e^x,\ 0\le x\le 1,$$

where

$$\alpha_3(x)=\int_0^x\alpha_2(t)dt.\ k_3=\frac{1}{2}\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_3(-1).\ k_2=\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right]-k_3.$$

$$k_1=\frac{1}{e-2}\left[\psi_2(2)-k_2(e-1)-k_3e-\int_0^1e^{1-t}\alpha_3(t)dt-\frac{1}{2}\int_1^2\left[\alpha_1(t)+\delta_2(t)\right]dt\right].$$

Thus, we have found the function $u_2(x,y)$ in the domain G_2 completely.

Now let's go to the area G_3. Passing in equations (19) $(i=2)$ and (19) $(i=3)$ to the limit at $y\to 0$, we find

$$\mu_2(x)=\tau_2''(x)-\omega_2(x),\ \mu_2(x)=\tau_2''(x)-\omega_3(x),\ -1\le x\le 0.$$

It follows from these equations $\omega_3(x)=\omega_2(x),\ -1\le x\le 0$. Changing the argument x to $x-y$, we have $\omega_3(x-y)=\omega_2(x-y),\ -1\le x-y\le 0$.

Further, passing in equations (19) $(i=3)$ and (18) to the limit at $x\to 0$ and excluding the function from the obtained equations $\mu_1(y)$, we obtain

$$\overline{\omega_1}(-y)=\omega_2(-y)+\left[\tau_4''(y)-\tau_4'(y)\right]e^{cy},\ (33)$$

where it should be $\omega_1(x-y)=\begin{cases}\overline{\omega_1}(x-y), & -1\le x-y\le 0,\\ \overline{\overline{\omega_1}}(x-y), & 0\le x-y\le 1.\end{cases}$

Consider the following auxiliary problem:

$$\begin{cases}u_{3xx}-u_{3yy}=\Omega_3(x-y)e^{-cy},\ (x,y)\in G_3,\\ u_3(x,0)=T_2(x),\ u_{3y}(x,0)=N_2(x),\ -2\le x\le 1,\\ u_3(-1,y)=\varphi_2(y),\ u_{3x}(-1,y)=\varphi_4(y),\ u_3(0,y)=\tau_4(y),\ 0\le y\le 1.\end{cases}$$

The solution of this problem that satisfies all the conditions of the same problem, except for the condition $u_{3x}(-1,y)=\varphi_4(y)$, will be sought in the form

$$u_3(x,y)=u_{31}(x,y)+u_{32}(x,y)+u_{33}(x,y),\ (34)$$

where is $u_{31}(x,y)-$ the solution of the problem

$$\begin{cases} u_{31xx} - u_{31yy} = 0, \\ u_{31}(x,0) = T_2(x),\ u_{31y}(x,0) = 0,\ -2 \le x \le 1, \\ u_{31}(-1,y) = \varphi_2(y),\ u_{31}(0,y) = \tau_4(y),\ 0 \le y \le 1; \end{cases} \quad (35)$$

$u_{32}(x,y)$ – the solution of the problem

$$\begin{cases} u_{32xx} - u_{32yy} = 0, \\ u_{32}(x,0) = 0,\ u_{32y}(x,0) = N_2(x),\ -2 \le x \le 1, \\ u_{32}(-1,y) = 0,\ u_{32}(0,y) = 0,\ 0 \le y \le 1; \end{cases} \quad (36)$$

$u_{33}(x,y)$ – the solution of the problem

$$\begin{cases} u_{33xx} - u_{33yy} = \Omega_3(x-y)e^{-cy}, \\ u_{33}(x,0) = 0,\ u_{33y}(x,0) = 0,\ -2 \le x \le 1, \\ u_{33}(-1,y) = 0,\ u_{33}(0,y) = 0,\ 0 \le y \le 1. \end{cases} \quad (37)$$

Using the continuation method, we find solutions to problems (35)-(37). They look like

$$u_{31}(x,y) = \frac{1}{2}\left[T_2(x+y) + T_2(x-y)\right], \quad (38)$$

$$u_{32}(x,y) = \frac{1}{2}\int_{x-y}^{x+y} N_2(t)\,dt, \quad (39)$$

$$u_{33}(x,y) = -\frac{1}{2}\int_0^y e^{-c\eta}d\eta \int_{x-y+\eta}^{x+y-\eta} \Omega_3(\xi-\eta)\,d\xi, \quad (40)$$

where

$$T_2(x) = \begin{cases} 2\varphi_2(-1-x) - \tau_2(-2-x),\ -2 \le x \le -1, \\ \tau_2(x),\ -1 \le x \le 0, \\ 2\tau_4(x) - \tau_2(-x),\ 0 \le x \le 1; \end{cases}$$

$$N_2(x) = \begin{cases} -\nu_2(-2-x),\ -2 \le x \le -1, \\ \nu_2(x),\ -1 \le x \le 0, \\ -\nu_2(-x),\ 0 \le x \le 1; \end{cases}$$

a $\Omega_3(x)$ is defined as follows: in the interval $-1 \le x \le 0$ it has the form $\Omega_3(x) = \omega_2(x)$, and in the intervals $-2 \le x \le -1$ and $0 \le x \le 1$ it is unknown.

The first two conditions of problem (37) are fulfilled automatically. Satisfying the third of the conditions of problem (37), after simplification, we obtain

$$\Omega_3(-1-y)\int_0^y e^{-c\eta}d\eta = -\int_0^y e^{-c\eta}\Omega_3(y-1-2\eta)\,d\eta. \quad (41)$$

Assuming in (40) $x \to 0$, after some transformations, we have

$$\omega_2(-y)\int_0^y e^{-c\eta}d\eta = -\int_0^y e^{-c\eta}\Omega_3(y-2\eta)\,d\eta.$$

Making a change of variables $y-2\eta=z$, from the last equality after long transformations, we find

$$\Omega_3(y)=2\omega_2'(-y)\int_0^y e^{-c\eta}d\eta-\omega_2(-y)\left[c\int_0^y e^{-c\eta}d\eta+3e^{-cy}\right]. \quad (42)$$

Substituting (38), (39), and (40) into (34), we have

$$u_3(x,y)=\frac{1}{2}\left[T_2(x+y)+T_2(x-y)\right]+\frac{1}{2}\int_{x-y}^{x+y}N_2(t)dt-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi-\eta)d\xi. \quad (43)$$

Differentiating this solution with respect to x, we have

$$u_{3x}(x,y)=\frac{1}{2}\left[T_2'(x+y)+T_2'(x-y)\right]+\frac{1}{2}\left[N_2(x+y)-N_2(x-y)\right]-$$

$$-\frac{1}{2}\int_0^y e^{-c\eta}\left[\Omega_3(x+y-2\eta)-\Omega_3(x-y)\right]d\eta. \quad (44)$$

Letting in (44) tend x to minus one, taking into account the condition $u_{3x}(-1,y)=\varphi_4(y)$ and equality (41), after some transformations, we find

$$\Omega_3(-1-y)=\left\{2\left[\tau_2''(y-1)+\nu_2'(y-1)-\varphi_2''(y)-\varphi_3'(y)\right]-\omega_2(y-1)+\right.$$

$$\left.+c\left[\tau_2'(y-1)+\nu_2(y-1)-\varphi_2'(y)-\varphi_3(y)\right]\right\}e^{cy},\ 0\le y\le 1.$$

Letting in (44) tend x to zero, taking into account (13) and (42), after some transformations we obtain the relation

$$\nu_4(y)=\tau_4'(y)+\beta_1(y),\ 0\le y\le 1, \quad (45)$$

where

$$\beta_1(y)=\tau_2'(-y)-\nu_2(y)-\omega_2(-y)\int_0^y e^{-c\eta}d\eta.$$

Now let's go to the area G_4. Passing in equations (19) $(i=2)$ and (20) $(i=4)$ to the limit at $y\to 0$, we find

$$\mu_3(x)=\tau_3''(x)-\omega_2(x),\ \mu_3(x)=\tau_3''(x)-\omega_4(x),\ 1\le x\le 2.$$

It follows from these equations $\omega_4(x)=\omega_2(x),\ 1\le x\le 2$. Changing the argument x to $x-y$, we have $\omega_4(x-y)=\omega_2(x-y),\ 1\le x-y\le 2$.

Further, passing in equations (19) $(i=4)$ and (18) to the limit at $x\to 1$, after some calculations, we obtain

$$\Omega_4(1-y)=\overline{\overline{\omega}}_1(1-y)-e^{-cy}\left[\tau_5''(y)-\tau_5'(y)\right]. \quad (46)$$

Now consider the following auxiliary problem:

$$\begin{cases} u_{4xx}-u_{4yy}=\Omega_4(x-y)e^{-cy},\ (x,y)\in G_4, \\ u_4(x,0)=T_3(x),\ u_{4y}(x,0)=N_4(x),\quad 0\le x\le 3, \\ u_4(2,y)=\varphi_1(y),\ u_4(1,y)=\tau_5(y),\ 0\le y\le 1. \end{cases}$$

The solution to this problem will be sought in the form

$$u_4(x,y)=u_{41}(x,y)+u_{42}(x,y)+u_{43}(x,y), \quad (47)$$

where is $u_{41}(x,y)$ – the solution of the problem

$$\begin{cases} u_{41xx} - u_{41yy} = 0, \\ u_{41}(x,0) = T_3(x),\ u_{41y}(x,0) = 0,\ 0 \le x \le 3, \\ u_{41}(2,y) = \varphi_1(y),\ u_{41}(1,y) = \tau_5(y),\ 0 \le y \le 1; \end{cases} \quad (48)$$

$u_{42}(x,y)$ – the solution of the problem

$$\begin{cases} u_{42xx} - u_{42yy} = 0, \\ u_{42}(x,0) = 0,\ u_{42y}(x,0) = N_3(x),\ 0 \le x \le 3, \\ u_{42}(2,y) = 0,\ u_{42}(1,y) = 0,\ 0 \le y \le 1; \end{cases} (49)$$

$u_{43}(x,y)$ – the solution of the problem

$$\begin{cases} u_{43xx} - u_{43yy} = \Omega_4(x-y)e^{-cy}, \\ u_{43}(x,0) = 0,\ u_{43y}(x,0) = 0,\ \ 0 \le x \le 3, \\ u_{43}(2,y) = 0,\ u_{43}(1,y) = 0,\ 0 \le y \le 1. \end{cases} \text{(fifty)}$$

Using the continuation method, we find solutions to problems (48)-(50). They look like

$$u_{41}(x,y) = \frac{1}{2}\left[T_3(x+y) + T_3(x-y)\right], (51)$$

$$u_{42}(x,y) = \frac{1}{2}\int_{x-y}^{x+y} N_3(t)\,dt, (52)$$

$$u_{43}(x,y) = -\frac{1}{2}\int_0^y e^{-c\eta}\,d\eta \int_{x-y+\eta}^{x+y-\eta} \Omega_4(\xi-\eta)\,d\xi, (53)$$

where

$$T_3(x) = \begin{cases} 2\varphi_1(x-2) - \tau_3(4-x),\ 2 \le x \le 3, \\ \tau_3(x),\ 1 \le x \le 2, \\ 2\tau_5(1-x) - \tau_3(2-x),\ 0 \le x \le 1; \end{cases}$$

$$N_3(x) = \begin{cases} -\nu_3(2-x),\ 0 \le x \le 1, \\ \nu_3(x),\ 1 \le x \le 2, \\ -\nu_3(4-x),\ 2 \le x \le 3; \end{cases}$$

a $\Omega_4(x)$ is defined as follows: in the interval $1 \le x \le 2$ it has the form $\Omega_4(x) = \omega_2(x)$, and in the intervals $0 \le x \le 1$ and $2 \le x \le 3$ it is unknown.

The first two conditions of problem (50) are satisfied automatically. Satisfying the third condition of problem (50), we find

$$\Omega_4(2+y) = -\omega_2(2-y) + 2\omega_2'(2-y)\int_0^y e^{-c\eta}\,d\eta. (54)$$

Assuming in (53) $x \to 1$, after some transformations, we have

$$2\Omega_4(1-y)\int_0^y e^{-c\eta}\,d\eta = -\int_{1-y}^{1+y} e^{-\frac{c}{2}(1+y-z)}\Omega_4(z)\,dz. (55)$$

Substituting (51), (52), and (53) into (47), we have

$$u_4(x,y)=\frac{1}{2}\left[T_3(x+y)+T_3(x-y)\right]+\frac{1}{2}\int_{x-y}^{x+y}N_3(t)\,dt-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_4(\xi-\eta)\,d\xi. \quad (56)$$

Differentiating this solution with respect to x, we have

$$u_{4x}(x,y)=\frac{1}{2}\left[T_3'(x+y)+T_3'(x-y)\right]+\frac{1}{2}\left[N_3(x+y)-N_3(x-y)\right]-$$

$$-\frac{1}{2}\int_0^y e^{-c\eta}\left[\Omega_4(x+y-2\eta)-\Omega_4(x-y)\right]d\eta. \quad (57)$$

Putting in (57) $x\to 1$ taking into account equalities (46) and (55), after some calculations and transformations, we arrive at the relation

$$\nu_5(y)=-\frac{1}{2}\tau_5'(y)-\frac{c+2}{4}\int_0^y e^{-\frac{c}{2}(y-\eta)}\tau_5'(\eta)\,d\eta+\beta_2(y),\ 0\le y\le 1, \quad (58)$$

where

$$\beta_1(y)=\tau_3'(1+y)+\nu_3(1+y)-\frac{1}{2}\int_0^y e^{-\frac{c}{2}(1+y-\eta)}\omega_2(\eta)\,d\eta-$$

$$-\frac{1}{2}\int_0^y e^{-\frac{c}{2}(y+\eta)}\overline{\overline{\omega_1}}(1-\eta)\,d\eta-\frac{1}{2}e^{\frac{c}{2}y}\nu_3(1).$$

Now let's move on to the area D_1. Passing in equation (18) to the limit at $y\to 0$, we find

$$\overline{\overline{\omega_1}}(x)=\tau_1''(x)-\nu_1(x),\ \ 0\le x\le 1,$$

where $\tau_1(x)$ and $\nu_1(x)$ are known functions.

Further, the solution of Eq. (18), which satisfies conditions (9) for $0\le x\le 1$, (12), and (15), is written as

$$u_1(x,y)=\int_0^y\tau_4(\eta)G_\xi(x,y;0,\eta)\,d\eta-\int_0^y\tau_5(\eta)G_\xi(x,y;1,\eta)\,d\eta+\int_0^1\tau_1(\xi)G(x,y;\xi,0)\,d\xi-$$

$$-\int_0^y e^{-c\eta}d\eta\int_0^\eta\overline{\omega_1}(\xi-\eta)G(x,y;\xi,\eta)\,d\xi-\int_0^y e^{-c\eta}d\eta\int_\eta^1\overline{\overline{\omega_1}}(\xi-\eta)G(x,y;\xi,\eta)\,d\xi.$$

Differentiating this solution with respect to x and tending x to zero and one, we obtain two more relations between the unknown functions $\tau_4(y)$, $\nu_4(y)$, $\tau_5(y)$ and $\nu_5(y)$. From these obtained two relations and (45), (58), after lengthy calculations, we arrive at a system of two Volterra integral equations of the second kind for unknown functions $\tau_4'(y)$ and $\tau_5'(y)$:

$$\tau_4'(y)+\int_0^y K_1(y,\eta)\tau_4'(\eta)\,d\eta+\int_0^y K_2(y,\eta)\tau_5'(\eta)\,d\eta=g_1(y),\ \ 0\le y\le 1, \quad (59)$$

$$\tau_5'(y)+\int_0^y K_3(y,\eta)\tau_5'(\eta)\,d\eta+\int_0^y K_4(y,\eta)\tau_4'(\eta)\,d\eta=g_2(y),\ \ 0\le y\le 1, \quad (60)$$

where $K_1(y,\eta)$, $K_2(y,\eta)$, $K_3(y,\eta)$, $K_4(y,\eta)$ and $g_1(y)$, $g_2(y)$ – well-known functions $K_1(y,\eta)$ and $K_3(y,\eta)$ have a weak singularity (of order $1/2$), the functions

$K_2(y,\eta)$, $K_4(y,\eta)$, $g_1(y)$ and $g_2(y)$ – are continuous, and

$$\left.\begin{matrix} G(x,y;\xi,\eta) \\ N(x,y;\xi,\eta) \end{matrix}\right\} = \frac{1}{2\sqrt{\pi(y-\eta)}} \sum_{n=-\infty}^{+\infty} \left\{ \exp\left[-\frac{(x-\xi-2n)^2}{4(y-\eta)} \right] \mp \exp\left[-\frac{(x+\xi-2n)^2}{4(y-\eta)} \right] \right\}$$

- Green's functions of the first and second boundary value problems for the Fourier equation . Solving the system of equations (59), (60), we find the functions $\tau_4'(y)$, $\tau_5'(y)$ and thus the functions $\nu_4(y)$, $\nu_5(y)$, $\overline{\omega_1}(-y)$, $\Omega_4(1-y)$, $T_2(x)$, $T_3(x)$, $N_2(x)$, $N_3(x)$, $u_1(x,y)$, $u_3(x,y)$.

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