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SELF -OSCILLATIONS OF A LINEAR VISCOELASTIC CANTILEVER ROD

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ANNOTATION

This article considers the problem of self-oscillations of physically linear viscoelastic rods in a gas flow, taking into account linear dependencies. A statement and a method for solving the problem of self-oscillations of a viscoelastic cantilever rod are presented. Numerical results are obtained.

Keywords: viscoelasticity, rod, self-oscillations, physical linearity, aerodynamic linearity, Bubnov-Galyorkin method, relaxation kernel, numerical method, integro-differential equation.

INTRODUCTION

Accounting for the hereditary effects of deformable materials is becoming increasingly necessary due to the fact that in most leading branches of modern technology, various elements and components of modern engineering structures are often operated under different conditions. The widespread use of composite materials in modern technology has led to the need to study the problems of optimal design of thin-walled structures with viscoelastic properties. In this regard, the hereditary theory of viscoelasticity attracts more and more attention of researchers. This is evidenced by the publication in recent years of a number of scientific papers that reflect the latest achievements in the theory of viscoelasticity.

The paper studies the linear flutter of a rod rigidly clamped at one end (cantilever). A one-dimensional rod model, taking into account the variability of width and thickness, allows to more correctly take into account the real shape of the rod.

MATHEMATICAL MODEL

Consider the problem of self-oscillations of a linear viscoelastic rod. The relationship between stresses σ and strains ε will be taken as [2, 17]:

$$\sigma = (1 - R^*) m_1 \varepsilon, \quad \varepsilon = u_x, \; u = -z w_x \qquad \text{(one)}$$

or

$$\sigma = -(1 - R^*) m_1 z w_{xx} \qquad (2)$$

where m $_{1 \text{ is an}}$ elastic constant, R (t) is the heredity kernel having weakly singular features of the Abel type, E is the modulus of elasticity .

Not taking into account also the influence of aerodynamic nonlinearity, according to the one-dimensional theory of gas, the gas pressure on the piston, the load will be taken in the form [4]:

$$q = \frac{\chi p_\infty}{c_\infty}\left[V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right]$$

where indicated $q = p - p_\infty$, $k = \dfrac{\chi p_\infty}{c_\infty}$

$$q = k\left[V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\right] \qquad (3)$$

Let us solve the problem of a self-oscillating process in a linear viscoelastic formulation, taking into account the physical and aerodynamic linearities. To this end, we construct a mathematical model for studying a viscoelastic rod in a gas flow, taking into account these linearities.

In this case, accepting the hypothesis of flat sections for the bending moment, we use the following formula [9]:

$$M_x = \int_{-h/2}^{h/2} b(x)\sigma_x z dz \qquad \text{(four)}$$

(2) put in (4) and get [2]:

$$M_x = m_1(1-R^*)J_2 w_{xx} \qquad (5)$$

where equal for beams of width b (x) and height h (x)

$$J_2 = \frac{b(x)h^3(x)}{12}.$$

Substituting (5) into the equilibrium equation , i.e. [9]

$$\frac{\partial^2 M_x}{\partial x^2} = m(x)\frac{\partial^2 w}{\partial t^2} + q(x,t) \qquad (6)$$

and passing to dimensionless coordinates and omitting the strokes, we have

$$(1-R^*)\frac{\partial}{\partial x^2}\left[J_2(x)w_{xx}\right] + F(x)w_{tt} + Pw_x + \gamma w_t = 0 \; (7)$$

where

$$W = h_0\overline{W},\, x = a\bar{x},\, t = t_1\bar{t},\, m = m_0 F(x),\, h(x) = h_0\overline{h(x)},\, b(x) = b_0\overline{b(x)},$$

$$J_2 = J_2^0 d(x), \quad J_2(x) = b(x)h^3(x), \quad J_2^0 = \frac{b_0 h_0^3}{12},$$

$$P = \frac{kVa^3}{m_1 J_2^{(0)}}, \quad t_1 = \sqrt{\left(m_0 a^4\right)/\left(m_1 J_2^{(0)}\right)}, \quad \gamma = \frac{kza^4}{m_1 J_2^{(0)} t_1}, \, F(x) = b(x)h(x)$$

h_0 - the value of the height of the rod at the ends, b_0 - the value of the width of the rod at the ends, m_0 - the value of the mass corresponding to a single variable section of the rod.

Solution method. We construct an approximate solution using the Bubnov-Galerkin method. We represent solution (7) in the form

$$w = \sum_{k=1}^{N} u_k(t)\varphi_k(x) \qquad \text{(eight)}$$

where $\varphi_k(x)$ - known, basic functions satisfying the given boundary conditions, $u_k(t)$ - unknown functions of time to be determined. N is the number of terms of the series in the expansion.

In this case, we obtain the following linear systems of ordinary integro-differential equations

$$\sum_{k=1}^{N}\left[a_{ki}\ddot{u}_k(t)+\gamma b_{ki}\dot{u}_k(t)+\omega_{ki}(1-R^*)u_k(t)+Pd_{ki}u_k(t)\right]=0,\ i=\overline{1,N} \qquad (9)$$

where

$$a_{ki}=\int_0^1 F(x)\varphi_k(x)\varphi_i(x)dx, \qquad b_{ki}=\int_0^1 \varphi_k(x)\varphi_i(x)dx,$$

$$\omega_{ki}=\int_0^1\left[d(x)\varphi_k''(x)\right]''\varphi_i(x)dx, \qquad d_{ki}=\int_0^1\varphi_k'(x)\varphi_i(x)dx,$$

Integration of the linear system (4) with the Rzhanitsyn–Koltunov kernel R (t)= A e ßt t $^{\alpha-1}$, A >0, β>0, 0< α<1 over a wide range of changes in the physical and mechanical parameters of the rod, will be performed by a numerical method based on analytical transformations [6]. According to this method, the numerical values of the desired functions $u_k(t_l)=u_{k,l}$ are found from the solution of the following recurrent system of linear algebraic equations

$$\sum_{k=1}^{N}\left[a_{ki}+\gamma\frac{\Delta t}{2}b_{ki}\right]u_{k,l}=\sum_{k=1}^{N}\left[(a_{ki}+\gamma t_l b_{ki})u_{k,0}+t_l a_{ki}\dot{u}_{k,o}\right]-\sum_{k=1}^{N}\sum_{i_1=1}^{l-1}\left[\gamma A_{i_1}b_{ki}u_{k,i_1}+A_{i_1}(t_l-t_{i_1})\cdot\right.$$

$$\left.\cdot\left(\omega_{ki}\left(u_{k,i_1}-\frac{A}{\alpha}\sum_{i_2=1}^{i_1}B_{i_2}e^{-\beta t_{i2}}u_{k,i_1-i_2+1}\right)+Pd_{ki}u_{k,i_1}\right)\right],\ i=\overline{1,N} \qquad \text{(ten)}$$

where

$$t_i=i\Delta t,\ B_1=\frac{\Delta t^\alpha}{2},\ B_{i_2}=\frac{\Delta t^\alpha\left[(i_2+1)^\alpha-(i_2-1)^\alpha\right]}{2},\ i_2=\overline{2,i_1-1}$$

$$B_{i_1}=\frac{\Delta t^\alpha\left[i_1^\alpha-(i_1-1)^\alpha\right]}{2},\ A_1=\frac{\Delta t}{2},\ A_i=\Delta t,\ i_1=\overline{2,i-1},\ i=1,2.....$$

The calculation was carried out for various rheological parameters and rod shapes in plan. The calculation was made for both ideally elastic and viscoelastic rods.

Beam functions [10] are taken as basis functions $\varphi_k(x)$

$$\varphi_k(x)=\sin\lambda_k x-sh\lambda_k x-\frac{\sin\lambda_k+sh\lambda_k}{\cos\lambda_k+ch\lambda_k}(\cos\lambda_k x-ch\lambda_k x);$$

$$\lambda_1=1.875,\quad \lambda_2=4.694,\quad \lambda_3=7{,}855,\quad \lambda_4=10{,}996,\ldots,\quad \lambda_k=\frac{\pi}{2}(2k-1) \qquad \text{(eleven)}$$

and for the initial conditions

$$u_k(0)=\int_0^1\alpha_0(x)\varphi_k(x)dx,\quad \dot{u}_k(0)=0 \text{ where } \alpha_0(x)=\left\{[x(1-x)]^4+\varphi_1(x)\right\}/10\,0$$

ANALYSIS AND CONCLUSION

An analysis of the results of physically linear problems given in the table shows that the critical velocity is determined by linear theory both in ideal elastic and viscoelastic formulations, and turns out to be only the upper limit of critical velocity for real structures.

Therefore, in order to investigate its influence on the critical speed, it is necessary to set a certain relationship between the shape of the rod in the plan and its rigidity, mass, that is, it is necessary to calculate for different α_1 and α_2 . For this purpose, a series of rods with a trapezoid shape of variable thickness is considered. Then the shape of the rod depends on the parameters α_1 and α_2, this parameter characterizes the narrowing of the rod. The value of the critical speed for various rods satisfying the boundary conditions is given in the table.

From the analysis of the calculation results, it follows that the value of the critical velocity in the elastic state of the linear $P_{cr.lin}$=67.69 and the difference with the viscous state is (at A=0.05) approximately 9.2% ($P_{cr.lin}$=61.4 3). By increasing the viscosity value, the value of the critical speed decreases. The influence of other parameters can be seen in the table. When the trapezoidal shape of the wing changes due to parameters α_1 and α_2, the influence of the shape is significant , the critical flutter velocity gets large values (Fig.03).

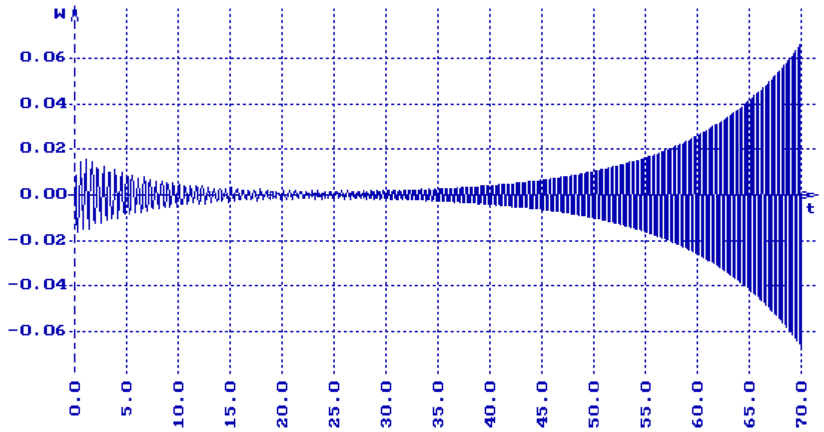
The designations in the table mean: $P_{kr.lin.}$ - critical speed. A is the viscosity coefficient; α, β are rheological parameters.

The cross section of the beam changes according to the law $b(x) = c - \alpha_1 x$; $h(x) = 1 - \alpha_2 x$; where c=5 .

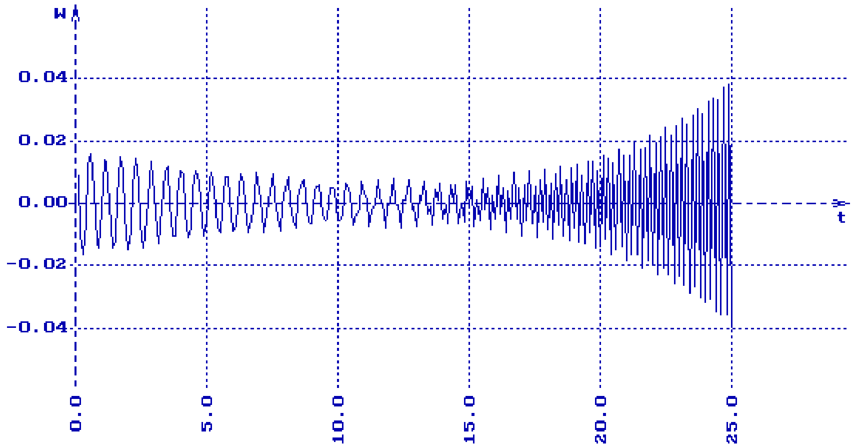
Table

N	BUT	α	β	α_1	α2 _	γ	$P_{cr.lin.}$
2	0.0	0.25	0.05	4.0	0.2	0.5	67.69
2	0.01 0.03 0.05 0.08 0.1	0.25	0.05	4.0	0.2	0.5	66.45 63.94 61.43 57.65 55.13
2	0.05	0.15 0.35 0.5	0.05	4.0	0.2	0.5	53.24 64.51 66.43
2	0.05	0.25	0.01 0.07 0.1	4.0	0.2	0.5	61.43 61.42 61.41
2	0.05	0.25	0.05	1.0 2.0 3.0	0.2	0.5	200.11 211.50 178.28
2	0.05	0.25	0.05	4.0	0.1 0.5 0.8	0.5	64.53 40.11 6.01
2	0.05	0.25	0.05	4.0	0.2	0.0 1.0 5.0 10.0	61 . 40 61 . 46 61 . 88 62 . 54

The figures show the change in the oscillation amplitude during flutter corresponding to the values of the critical velocity .

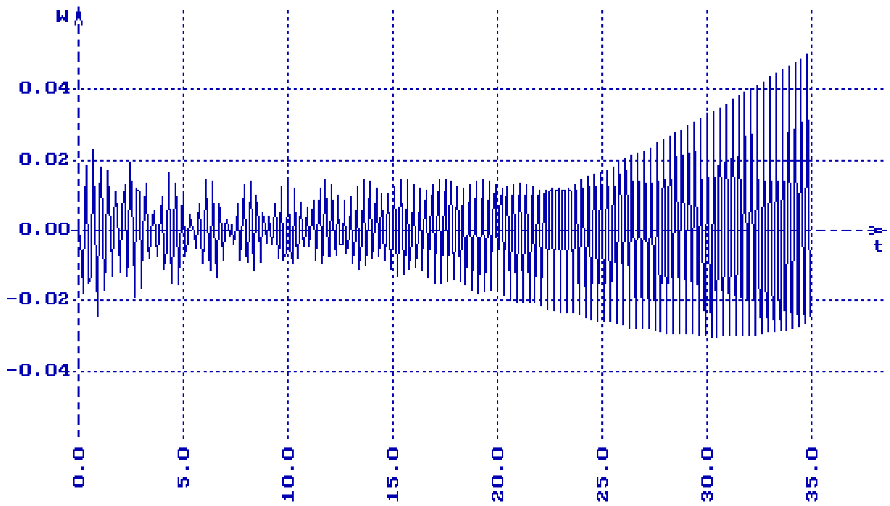


Rice. 01. A=0.0, α=0.25, h =0. 1 , β =0.05, α_1=4.0, α_2=0.2
N \u003d 2, speed: P_{cr} \ u003d 67 . 69

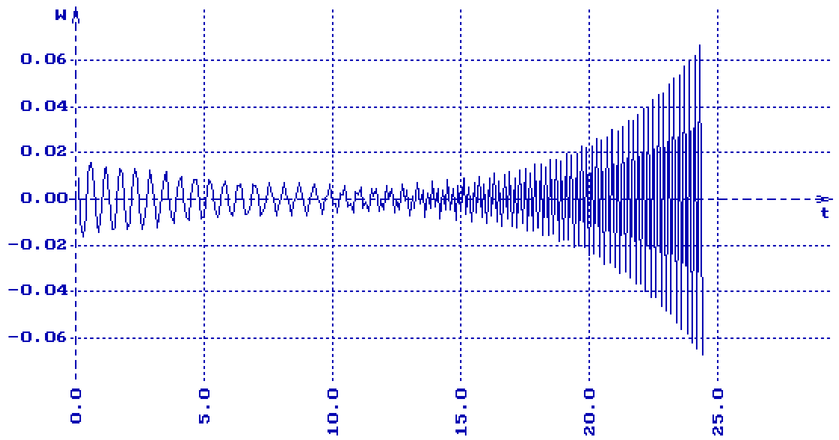


Rice. 02. A=0.05, α=0.25, h =0. 1 , β =0.05, α_1=4.0, α_2=0.2
N \u003d 2, speed: P_{cr} \ u003d 61 . 43

Stem Shape Effects:



Rice. 03. A=0.05, α=0.25, h =0. 1 , β =0.05, α_1= 2 .0, α_2=0.2
N \u003d 2, speed: P_{cr} \ u003d 211 . fifty



Rice. 13. A=0.05, α=0.25, h =0. 1 , β =0.05, α_1=4.0, α_2=0. 5
N \u003d 2, speed: P_{cr} \ u003d 40 . eleven

BIBLIOGRAPHY

1. Ivanov S.P., Ivanov O.G., Ivanova A.S. Oscillations and stability of rods from physically nonlinear materials // Stroitelnaya mekhanika inzhenernykh konstruktsii i sooruzheniy, 2011, 3, pp. 3-6.
2. Badalov F.B. Power series method in the nonlinear theory of viscoelasticity. Tashkent, "FAN" 1980. 221s.
3. Bolotin V.V. Non-conservative problems of the theory of elastic stability. Fizmatgiz M. 1991. 339s.
4. Bolotin V.V. On critical speeds in the nonlinear theory of aeroelasticity. Scientific reports Higher School "Mechanical Engineering and Instrumentation". 1958. No. 3.
5. Bolotin V.V. Nonlinear flutter of plates and shells. Engineering Collection. T.28. 1960.
6. Volmir A.S. Stability of deformable systems. Ed. "The science". The main edition of physical and mathematical literature. Moscow, 1967
7. Ilyushin A.A. The law of plane sections in aerodynamics at high supersonic speeds. PMM. 1956 V.20 No. 6 p. 733-755.
8. Algazin S.D., Kiyko I.A. Flutter of plates and shells. M.: Nauka, 2006.
9. Volmir A.S. Nonlinear dynamics of plates and shells. Ed. "The science". The main edition of physical and mathematical literature. Moscow, 1972
10. Babakov I.M. Theory of vibrations. Ed. "The science". The main edition of the physical and mathematical literature. Moscow, 1968.
11. Badalov FB Methods for solving integral and integro-differential equations of the hereditary theory of viscoelasticity. Tashkent, "Mekhnat", 1987. 269s.
12. Mansurov, M.; Abdikarimov, R.; Mirsaidov, M. Self-oscillatory process of a viscoelastic elongated plate ; 2022; Construction of Unique Buildings and Structures; 100 Article No 10003. doi: 10.4123/CUBS.100.3 .
13. Abdikarimov RA, Mansurov MM, Akbarov UY Numerical study of a flutter of a viscoelastic rigidly clamped rod with regard for the physical and aerodynamic nonlinearities. Bulletin of the Russian State University for the Humanities. Series "Informatics. information security. Maths. Science Magazine. 2019. 3. Pp. 94–106. DOI:10.28995/2686-679X-2019-3-94-106.

14. Abdikarimov R.A., Akbarov U.Y., Pulatov Sh.Y., Mansurov M.M. Flutter of a viscoelastic rod hinged at the ends. Scientific journal FerPI. F., 3/2020, 9-14 p.
15. Rustamkhan A. Abdikarimov and Mukhsin M. Mansurov Flutter of a viscoelastic rigidly restrained bar with account for nonlinearities. AIP Conference Proceedings 2637, 030003 (2022); https://doi.org/10.1063/5.0121701
16. MM Mansurov, UY Akbarov Flutter of a linear viscoelastic rod, freely supported at the ends. Scientific Bulletin of NamSU. no. 3, 2021.pp. 36–43
17. Eshmatov Kh.E., Nasretdinova Sh.S. Mathematical modeling of nonlinear problems in the dynamics of viscoelastic systems. - Tashkent, "Moliya", 2000. 108 p.
18. Mamazhonov, M., & Shermatova, KM (2017). ON A BOUNDARY-VALUE PROBLEM FOR A THIRD-ORDER PARABOLIC-HYPERBOLIC EQUATION IN A CONCAVE HEXAGONAL DOMAIN. Bulletin KRASEC. Physical and Mathematical Sciences , 16 (1), 11-16.
19. Mamajonov, M., & Shermatova, H. M. (2017). On a boundary value problem for a third-order equation of parabolic-hyperbolic type in a concave hexagonal region. Vestnik KRAUNTS. Physical and Mathematical Sciences , (1 (17), 14-21.
20. Mamazhanov M., Shermatova H.M., Mamadalieva H.B. On a boundary value problem for a third-order parabolic-hyperbolic type equation in a concave hexagonal region. Actual scientific research in the modern world. ISCIENCE . IN . UA , Pereyaslav-Khmelnitsky, 2017, issue 2(22), pp. 148-151.