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TOPICS : GAUSS 'S THEOREM. INTEGRAL EXPRESSION OF THE HYPERGEOMETRIC FUNCTION ACCORDING TO THE DALANBER PRINCIPLE

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ABSTRACT

The article considers boundary value problems for hypergeometric functions and their differential equations, problems in the integral form of hypergeometric functions. The hypergeometric function of Gauss, the Dalanber principle was used .

Keywords; hypergeometric function, differential equations, integral form, d'Alembert principle

HYPERGEOMETRIC EQUATION

Basic Definitions

Type equation

$$x(1-x)y''+\left[c-(a+b+1)x\right]y'-aby=0 \qquad (1)$$

is called *the hypergeometric equation* or *the Gauss equation,* where a, b, c are three arbitrary parameters that take real or complex values. Two of them: a and b symmetrically participate in the equation.

$$\sum_{n=0}^{\infty} M\left|\frac{x}{x_0}\right|^2 = M + M\left|\frac{x}{x_0}\right| + M\left|\frac{x}{x_0}\right|^2 + \ldots + M\left|\frac{x}{x_0}\right|^n + \ldots$$

solution of the equation

$$y=\sum_{n=0}^{\infty} A_n x^n \qquad (2)$$

looking for in the form of a power series. From this

$$y'=\sum_{n=1}^{\infty} nA_n x^{n-1}$$

or

$$y'=\sum_{n=0}^{\infty}(n+1)A_{n+1}x^n ,$$

$$y''=\sum_{n=1}^{\infty} n(n+1)A_{n+1}x^{n-1}=\sum_{n=0}^{\infty}(n+1)(n+2)A_{n+2}x^n .$$

the values of these derivatives and y (1) into the equation. In this case

$$\sum_{n=0}^{\infty} x(1-x)(n+1)(n+2)A_{n+2}x^n +$$

$$+\sum_{n=0}^{\infty}\left[c-(a+b+1)x\right](n+1)A_{n+1}x^n-\sum_{n=0}^{\infty}abA_nx^n=0.$$

unknown A_1 constants x, ..., , ... we use the method of uncertain coefficients, on the A_n basis of which the coefficients in front of the same levels should be set equal to zero. x^n equating the common coefficients before that to zero

$$-(n-1)nA_n+n(n+1)A_{n+1}-n(a+b+1)A_n+c(n+1)A_{n+1}-abA_n=0$$

make up an equation. From this

$$A_{n+1}=\frac{(n+a)(n+b)}{(n+1)(c+n)}A_n$$

we get a recursive formula.

Here we $A_0=1$ also $c\neq 0,-1,-2,\dots,-n,\dots$ think that Let's determine the first particular A_n solution of the $F(a,b,c,x)$ hypergeometric equation (1) and substitute y_1 the found values of the coefficients in line (2). In this case

$$y_1=F(a,b,c,x)=1+\sum_{n=1}^{\infty}\frac{(a)_n(b)_n}{(c)_n(1)_n}x^n. \qquad (3)$$

Here

$$(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}=a(a+1)\dots(a+n-1),$$

$$(a)_0=1 \qquad n=1,2,3,\dots,$$

privately, $(1)_n=n!$

(3) is a *hypergeometric series, and* the function that is the sum of this series is called $F(a,b,c,x)$ *a hypergeometric function* .

According to d'Alembert's principle,

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|=\lim_{n\to\infty}\left|\frac{(a+n)(b+n)}{(n+1)(c+n)}x\right|=|x|.$$

Therefore, series (3) $|x|<1$ absolutely converges $|x|>1$ and diverges. $x=1$ for , if $c-a-b>0$, (3) the series is absolutely convergent, if $c-a-b\leq 0$, then it is $c-a-b>0$ divergent $x=-1$, $-1<c-a-b\leq 0$ and if it $c-a-b\leq -1$ exists, then it will be distant.

If (3) in the formula $b=c$ if

$$(a)_n=(-1)^n(-a)(-a-1)\dots(-a-n+1)=(-1)^n\binom{-a}{n}n!$$

based

$$F(a,b,b;x)=1+\sum_{n=1}^{\infty}(-1)^n\binom{-a}{n}x^n=(1-x)^{-a}$$

a binomial series is formed.

If $a=1$, then $b=c$ formula (3) is

$$F(1,b,b;x)=1+\sum_{n=1}^{\infty}x^n=\frac{1}{1-x}$$

will look like, $a=1$e $b=c$. the hypergeometric series becomes a geometric progression, which is why it is called the hypergeometric series.

(3) into equation (1) to find the second particular solution of equation (3)

$$y=x^{\rho}\eta$$

we will replace. Then equation (1) is written in the following form:

$$x(1-x)\eta''+\left[(c+2\rho)-(a+b+1+2\rho)x\right]\eta'-$$

$$-\left[ab+\rho(a+b+\rho)-\frac{\rho(\rho+c-1)}{x}\right]\eta=0$$

was an equation like equation $\rho=0$(1), or $\rho=1-c$ should be. $\rho=1-c$ Existence

$$x(1-x)\eta''+\left\{(2-c)-\left[(a-c+1)+(b-c+1)+1\right]x\right\}\eta'-$$

$$-(a-c+1)(b-c+1)\eta=0$$

we get the $\rho=1-c$ replacement equation (c 1 a) b when $y=x^{\rho}\eta$

$$a-c+1. \quad b-c+1. \quad 2-c$$

needs to be replaced. Therefore, this equation (y_1 1) is not linearly related to

$$y_2=x^{1-c}F(a-c+1,b-c+1,2-c;x)$$

will have a solution. In the same time, y_2

$$2-c\neq 0,-1,-2,\ldots,-n,\ldots$$

makes sense only when Thus, the general solution of equation (1) can be written in the following form:

$$y=C_1F(a,b,c;x)+C_2x^{1-c}F(a-c+1,b-c+1,2-c;x),$$

where C_1 and C_2 are arbitrary constants.

If the hypergeometric function is symmetric and one a of b the parameters is a negative integer n equal to - , then the hypergeometric series (3) is interrupted and n becomes a -degree polynomial.

If $a=-n_1$, $b=-n_2$, where $n_1>0$, $n_2>0$ are integers, then the hypergeometric series becomes a polynomial whose degree n_1 is n_2 equal to the smallest of the numbers . (3) as a result of differentiating the series as follows

$$F(a,b,a;x)=\frac{ab}{c}F(a+1,b+1,c+1;x)$$

create a formula.

(3) by x^a, x^b or x^{c-1} then differentiate it, then the following formulas will be obtained:

$$\frac{d}{dx}\left[x^aF(a,b,c;x)\right]=ax^{a-1}F(a+1,b,c;x),$$

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$$\frac{d}{dx}\left[x^{b}F(a,b,c;x)\right]=bx^{b-1}F(a,b+1,c;x), \qquad (4)$$

$$\frac{d}{dx}\left[x^{c-1}F(a,b,c;x)\right]=(c-1)x^{a-1}F(a,b,c-1;x).$$

Integral expression of the hypergeometric function

(3) line

$$(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}$$

given equality, this

$$F(a,b,c;x)=1+\sum_{n=1}^{\infty}\frac{\Gamma(c)\Gamma(a+n)\Gamma(b+n)}{\Gamma(a)\Gamma(b)\Gamma(c+n)n!}x^n=$$

$$=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\left[\frac{\Gamma(a)\Gamma(b)}{\Gamma(c)}+\sum_{n+1}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)n!}x^n\right]=$$

$$=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)n!}x^n$$

write in the form.

Based on formula (3).

$$B(b+n,c-b)=\frac{\Gamma(b+n)\Gamma(c-b)}{\Gamma(c+n)}$$

Since , the previous equality

$$F(a,b,c;x)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)}{n!}x^nB(b+n,c-b)$$

is written in the form or according to (2. 1.3).

$$F(a,b,c;x)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)\Gamma(c-b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)}{n!}x^n\int_0^1 t^{n+b-1}(1-t)^{c-b-1}dt\,.$$

how does the integral here n converge for all values

$b>0$ or $c-b>0$ ($c>b>0$ (5)

conditions must be met.

This is the previous equation

$$F(a,b,c;x)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)}{n!\Gamma(a)}\int_0^1 t^{n+b-1}(1-t)^{c-b-1}x^n dt=$$

$$=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}\left[\sum_{n=0}^{\infty}\frac{(a)_n}{n!}(xt)^n\right]dt$$

write in the form. Since the sum under the integral $(1-xt)^{-a}$ is an expansion into an infinite series of the function

$$F(a,b,c;x)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-xt)^{-a}\,dt \qquad (6)$$

formula . This is an integral expression of the hypergeometric function.

(5) conditions are the same

$a-b-c>0$

conditionally replaceable. If $a<0$ there is, $-a>0$ then it will be, and by adding the inequality of this inequality (5) with the second one, $a-b-c>0$ we obtain the inequality; if $a>0$ so, then from this inequality we have (5), which is stronger than the second $c-b>a$ inequality.

Let us calculate the value of the hypergeometric function . Since the $x=1$ integral in formula $b>0$ (6) is a smooth approximation for $x\to 1$, $c>0$ and we pass to $|x|<1$ the limit:

$$\lim_{x\to l}F(a,b,c;x)=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\lim_{x\to 1}\int_0^1\left[t^{b-1}(1-t)^{c-b-1}(1-xt)^{-a}\right]dt=$$

$$=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1\lim_{x\to 1}\left[t^{b-1}(1-t)^{c-b-1}(1-xt)^{-a}\right]dt=$$

$$=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 t^{b-1}(1-t)^{c-b-a-1}\,dt=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}B(b,c-b-a)=$$

$$=\frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\frac{\Gamma(b)\Gamma(c-b-a)}{\Gamma(c-a)}=\frac{\Gamma(c)\Gamma(c-b-a)}{\Gamma(c-a)\Gamma(c-b)}.$$

So,

$$\lim_{x\to 1}F(a,b,c;x)=F(a,b,c;1)=\frac{\Gamma(c)\Gamma(c-b-a)}{\Gamma(c-a)\Gamma(c-b)}.$$

If (6) in the integral in the formula

$$t=\frac{1-s}{1-xs} \text{ or } s=\frac{1-t}{1-tx}$$

If we make a substitution, then the integral will be written in the following form:

$$\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-xt)^{-a}\,dt=$$

$$=(1-x)^{c-a-b}\int_0^1 s^{c-b-1}(1-s)^{b-1}(1-xs)^{-(c-a)}\,ds=$$

$$=\frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)}(1-x)^{(c-a-b)}F(c-a,c-b,c;x).$$

So,

$$F(a,b,c;x)=(1-x)^{(c-a-b)}F(c-a,c-b,c;x).$$

This equality is called the auto-conversion formula.

(6) by replacing the variable $t=1-s$ by the formula in the integral

$$\int_0^1 t^{a-1}(1-t)^{c-a-1}(1-xt)^{-b}\,dt=$$

$$=(1-x)^{-b}\int_0^1 s^{c-a-1}(1-s)^{a-1}\left(1-\frac{x}{x-1}s\right)^{-b}ds$$

make up an equation. Hence, if we take into account (6),

$$F(a,b,c;x)=(1-x)^{-b}F\left(c-a,b,c;\frac{x}{x-1}\right)$$

formula is formed.

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