ON ONE BOUNDARY PROBLEM FOR ONE PARABOLIC-HYPERBOLIC EQUATION OF THE THIRD ORDER IN A QUADRANGULAR DOMAIN WITH TWO LINES TYPE CHANGES

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ANNOTATION

In this thesis, two types are in a rectangular region with a variation line $\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}+c\right)(Lu)=0$ for a third-order parabolic-hyperbolic equation in the form a single boundary value problem is posed and studied.

Keywords: parabolic-hyperbolic type, boundary value problem, type change line, equation solution, integral equation, differential equation, quadrangular sphere.

INTRODUCTION

The study of various problems for equations of the third and higher orders of the parabolic-hyperbolic type began in the 1970s and 1980s . Such problems were studied mainly by T. D. Dzhuraev and his students (for example, see [1] , [2]).

At present, the study of various boundary value problems for equations of the third and higher orders of the parabolic-hyperbolic type is being developed in a broad sense. (for example, see [3] - [5]).

FORMULATION OF THE PROBLEM

In the plane xOy region G, consider the equation

$$\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}+c\right)(Lu)=0, \text{ (one)}$$

where $c\in R$, $Lu\equiv\begin{cases}L_1u\equiv u_{xx}-u_y, & (x,y)\in G_1,\\ L_iu\equiv u_{xx}-u_{yy}, & (x,y)\in G_i\ (i=2,3),\end{cases}$ $G=G_1\cup G_2\cup G_3\cup J_1\cup J_2$, G_1- rectangle with vertices at points $A(0,0)$, $B(1,0)$, $B_0(1,1)$, $A_0(0,1)$; G_2-triangle with vertices at points $A(0,0)$, $A_0(0,1)$, $D(-1,0)$; G_3-triangle with vertices at points $B(1,0)$, $B_0(1,1)$, $E(2,0)$; J_1- open segment with vertices at points $A(0,0)$and $A_0(0,1)$; J_2- open segment with vertices at points $B(1,0)$ and $B_0(1,1)$, i.e. $G-$ quadrilateral with vertices at points $D(-1,0)$, $A_0(0,1)$, $B_0(1,1)$, $E(2,0)$.

In this area, there is

A task $M^1_{1(-1)c}$. Find a function $u(x,y)$ that is 1) continuous in a closed region $\overline{G}$ and in the area $G\setminus J_1\setminus J_2$ has continuous derivatives involved in equation (1), and u_x, u_y and u_{yy} are continuous in G up to the part of the boundary of the region G specified in the boundary conditions ; 2) satisfies equation (1) in the region $G\setminus J_1\setminus J_2$; 3) satisfies the following boundary conditions:

$$u(x,0)=f_1(x),\ 0\le x\le 1, \quad (2)$$

$$u_y(x,0)=f_2(x),\ 0\le x\le 1, \quad (3)$$

$$u(x,0)=f_3(x),\ -1\le x\le 0, \quad (4)$$

$$u_y(x,0)=f_4(x),\ -1\le x\le 0, \quad (5)$$

$$u_{yy}(x,0)=f_5(x),\ -1< x< 0, \quad (6)$$

$$u(x,0)=f_6(x),\ 1\le x\le 2, \quad (7)$$

$$u_y(x,0)=f_7(x),\ 1\le x\le 2, \quad (8)$$

$$u_{yy}(x,0)=f_8(x),\ 1< x< 2 \quad (9)$$

and 4) satisfies the following continuous gluing conditions :

$$u(-0,y)=u(+0,y)=\tau_1(y),\ 0\le y\le 1, \quad \text{(ten)}$$

$$u_x(-0,y)=u_x(+0,y)=\nu_1(y),\ 0\le y\le 1, \quad \text{(eleven)}$$

$$u_{xx}(-0,y)=u_{xx}(+0,y)=\mu_1(y),\ 0< y< 1, \quad (12)$$

$$u(1-0,\ y)=u(1+0,\ y)=\tau_2(y),\ 0\le y\le 1, \quad (13)$$

$$u_x(1-0,\ y)=u_x(1+0,y)=\nu_2(y),\ 0\le y\le 1, \quad \text{(fourteen)}$$

$$u_{xx}(1-0,y)=u_{xx}(1+0,y)=\mu_2(y),\ 0< y< 1, \quad \text{(fifteen)}$$

where f_i $\left(i=\overline{1,9}\right)$ are given sufficiently smooth functions, and τ_j, ν_j, μ_j $(j=1,2)$ are still unknown fairly smooth functions.

Theorem. Let $f_1\in C^3[0,1]$, $f_2\in C^2[0,1]$, $f_3\in C^3[-1,0]$, $f_4\in C^2[-1,0]$, $f_5\in C^1[-1,0]$, $f_6\in C^3[1,2]$, $f_7\in C^2[1,2]$, $f_8\in C^1[1,2]$ and the matching conditions are met $\tau_1(0)=f_1(0)=f_3(0)$, $\nu_1(0)=f_2(0)=f_4(0)$, $\tau_2(0)=f_1(1)=f_6(1)$, $\nu_2(0)=f_2(1)=f_7(1)$, then the task $M^1_{1(-1)c}$ admits a unique solution .

To prove this theorem, we introduce the notation $u(x,y)=u_i(x,y)$, $(x,y)\in G_i$ $(i=1,2,3)$. Then equation (1) can be rewritten as

$$u_{1xx}-u_{1y}=\omega_1(x+y)e^{-cy}, \quad (16)$$

$$u_{ixx}-u_{iyy}=\omega_i(x+y)e^{-cy}\ (i=2,3), \quad (17)$$

where $\omega_i(x+y)$ $(i=1,2,3)$ – unknown yet sufficiently smooth functions.

Sachala task $M^1_{1(-1)c}$ consider in the area G_3. Passing in equation (17) ($i=3$) to the limit at $y\to 0$, due to conditions (7) and (9) we find

$$\omega_3(x)=f_6''(x)-f_8(x),\ 1\le x\le 2.$$

In this equality, changing x on $x+y$, we have

$$\omega_3(x+y)=f_6''(x+y)-f_8(x+y),\ 1\le x+y\le 2.$$

Now let's look at the following auxiliary problem：

$$\begin{cases} u_{3xx}-u_{3yy}=\Omega_3(x+y)e^{-cy},\ (x,y)\in G_3, \\ u_3(x,0)=F_6(x),\ u_{3y}(x,0)=F_7(x),\ 0\le x\le 2, \\ u_3(1,y)=\tau_2(y),\ u_{3x}(1,y)=\nu_2(y),\ 0\le y\le 1. \end{cases} \text{(eighteen)}$$

The solution of equation (1 8) that satisfies all conditions except the condition $u_{3x}(1,y)=\nu_2(y)$, we will search in the form

$$u_3(x,y)=u_{31}(x,y)+u_{32}(x,y)+u_{33}(x,y) \quad (19)$$

where $u_{31}(x,y)-$ the solution of the problem

$$\begin{cases} u_{31xx}-u_{31yy}=0, \\ u_{31}(x,0)=F_6(x),\ u_{31y}(x,0)=0,\ 0\le x\le 2, \\ u_{31}(1,y)=\tau_2(y),\ 0\le y\le 1, \end{cases} \text{(twenty)}$$

$u_{32}(x,y)-$ the solution of the problem

$$\begin{cases} u_{32xx}-u_{32yy}=0, \\ u_{32}(x,0)=0,\ u_{32y}(x,0)=F_8(x),\ 0\le x\le 2, \\ u_{32}(1,y)=0,\ 0\le y\le 1, \end{cases} (21)$$

$u_{33}(x,y)-$ the solution of the problem

$$\begin{cases} u_{33xx}-u_{33yy}=\Omega_3(x+y)e^{-cy},\ (x,y)\in G_3, \\ u_{33}(x,0)=0,\ u_{33y}(x,0)=0,\ 0\le x\le 2, \\ u_{33}(1,y)=0,\ 0\le y\le 1. \end{cases} \quad (22)$$

Here the functions $F_6(x)$, $F_7(x)$ and $\Omega_3(x+y)$ are defined as follows：when $1\le x\le 2$ the functions $F_6(x)$ wa $F_7(x)$ are known: $F_6(x)=f_6(x)$, $F_7(x)=f_7(x)$, and for $0\le x\le 1$ they are still unknown . Function $\Omega_3(x+y)$ in between $1\le x+y\le 2$ known, i.e. $\Omega_3(x+y)=\omega_3(x+y)$, and in between $0\le x+y\le 1$ she is still unknown .

The solution to problem (20) that satisfies the first two conditions is written as

$$u_{31}(x,y)=\frac{1}{2}[F_6(x+y)+F_6(x-y)]. \quad (23)$$

Substituting (2 3) into the third condition of the problem (20), we get

$$F_6(1-y)=2\tau_2(y)-f_6(1+y),\ 0\le y\le 1 \quad (2\ 4)$$

AT (24) changing $1-y$ to x, we find

$$F_6(x)=2\tau_2(1-x)-f_6(2-x),\ 0\le x\le 1$$

Then we have

$$F_6(x)=\begin{cases} 2\tau_2(1-x)-f_6(2-x),\ 0\le x\le 1, \\ f_6(x),\ 1\le x\le 2. \end{cases}$$

Now let's write a solution to the problem (2 1) that satisfies the first two conditions：

$$u_{32}(x,y)=\frac{1}{2}\int_{x-y}^{x+y}F_7(t)dt\,. \quad (25)$$

Substituting (2 5) into the third condition of the problem (21), we obtain

$$F_7(1-y)=-f_7(1+y),\ 0\le y\le 1. \quad (2\ 6\ )$$

AT (26) changing $1-y$ on x, we find

$$F_7(x)=-f_7(2-x),\ 0\le x\le 1.$$

So ,

$$F_7(x)=\begin{cases}-f_7(2-x),\ 0\le x\le 1,\\ f_7(x),\ 1\le x\le 2.\end{cases}$$

Finally , let's write down the solution of the problem (2 2) that satisfies the first two conditions :

$$u_{33}(x,y)=-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi+\eta)d\xi\,. \quad (27)$$

Substituting (2 7) into the third condition of problem (2 2) , we obtain

$$\int_0^y e^{-c\eta}\Omega_3(1-y+2\eta)d\eta=-\omega_3(1+y)\int_0^y e^{-c\eta}d\eta\,. \quad (2\ 8\ )$$

In the integral on the left parts of equality (2 8) , making a replacement $1-y+2\eta=1-z$, we get

$$\int_{-y}^{y} e^{-\frac{c}{2}(y-z)}\Omega_3(1-z)dz=-2\omega_3(1+y)\int_0^y e^{-c\eta}d\eta\,. \quad (29)$$

Differentiating equalities (29) and taking into account equality (29) itself, after some calculations, we find

$$\Omega_3(1-y)=-[2\omega_3'(1+y)+c\omega_3(1+y)]\int_0^y e^{-c\eta}d\eta-3\omega_3(1+y)e^{-cy}\,.$$

Now substituting (2 3), (2 5) and (2 7) into (19), we have

$$u_3(x,y)=\frac{1}{2}[F_6(x+y)+F_6(x-y)]+\frac{1}{2}\int_{x-y}^{x+y}F_7(t)dt-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi+\eta)d\xi\,. \quad \text{(thirty)}$$

Differentiating (30) with respect to x, we find

$$u_{3x}(x,y)=\frac{1}{2}[F_6'(x+y)+F_6'(x-y)]+\frac{1}{2}[F_7(x+y)-F_7(x-y)]-$$

$$-\frac{1}{2}\int_0^y e^{-c\eta}[\Omega_3(x+y)-\Omega_3(x-y+2\eta)]d\eta\,. \quad (31)$$

Passing in (3 1) to the limit at $x\to 1$ and taking into account (2 4), (2 6) and (2 8) , we arrive at the relation

$$\nu_2(y)=-\tau_2'(y)+\beta_1(y),\ 0\le y\le 1, \quad (32)$$

where

$$\beta_1(y)=f_6'(1+y)+f_7(1+y)-\omega_3(1+y)\int_0^y e^{-c\eta}d\eta\,.$$

Now the task $M^1_{1(-1)c}$ consider in the area G_2. Passing in equation (3.1.17) ($i=2$) to the limit at $y\to 0$, due to (4) and (6) , we obtain

$$\omega_2(x)=f_3''(x)-f_5(x),\ -1\le x\le 0.$$

In this equality, changing x on $x+y$, we find

$$\omega_2(x+y)=f_3''(x+y)-f_5(x+y),\ -1\le x+y\le 0.$$

Now consider the following auxiliary problem :

$$\begin{cases} u_{2xx}-u_{2yy}=\Omega_2(x+y)e^{-cy},\ (x,y)\in G_2,\\ u_2(x,0)=F_3(x),\ u_{2y}(x,0)=F_4(x),\ -1\le x\le 1,\\ u_2(0,y)=\tau_2(y),\ u_{2x}(0,y)=\nu_2(y),\ 0\le y\le 1. \end{cases} \quad (33)$$

Solution of problem (33) that satisfies all the conditions of this problem except $u_{2x}(0,y)=\nu_2(y)$, we will search in the form

$$u_2(x,y)=u_{21}(x,y)+u_{22}(x,y)+u_{23}(x,y), \quad (34)$$

here $u_{21}(x,y)-$the solution of the problem

$$\begin{cases} u_{21xx}-u_{21yy}=0,\\ u_{21}(x,0)=F_3(x),\ u_{21y}(x,0)=0,\ -1\le x\le 1,\\ u_{21}(0,y)=0,\ 0\le y\le 1, \end{cases} \quad (35)$$

$u_{22}(x,y)-$the solution of the problem

$$\begin{cases} u_{22xx}-u_{22yy}=0,\\ u_{22}(x,0)=0,\ u_{22y}(x,0)=F_4(x),\ -1\le x\le 1,\\ u_{22}(0,y)=0,\ 0\le y\le 1, \end{cases} \quad (36)$$

$u_{23}(x,y)-$the solution of the problem

$$\begin{cases} u_{23xx}-u_{23yy}=\Omega_2(x+y)e^{-cy},\ (x,y)\in G_2,\\ u_{23}(x,0)=0,\ u_{23y}(x,0)=0,\ -1\le x\le 1,\\ u_{23}(0,y)=0,\ 0\le y\le 1. \end{cases} \quad (37)$$

Here $F_3(x)$, $F_4(x)$ and $\Omega_2(x-y)$ are defined as follows :

In the interim $-1\le x\le 0$ functions $F_3(x)$ and $F_4(x)$ known , i.e. $F_3(x)=f_3(x)$, $F_4(x)=f_4(x)$, and in the interval $0\le x\le 1$ they are still unknown . In the interval $-1\le x+y\le 0$, the function $\Omega_2(x+y)$ is known, i.e. $\Omega_2(x+y)=\omega_2(x+y)$, and in between $0\le x+y\le 1$ she is still unknown .

Let's write down the solution of problem (35) that satisfies the first two conditions of this problem :

$$u_{21}(x,y)=\frac{1}{2}[F_3(x+y)+F_3(x-y)]. \quad (38)$$

Substituting (38) into the third condition of problem (35) , we obtain

$$F_3(y)=2\tau_1(y)-f_3(-y),\ 0\le y\le 1. \quad (39)$$

Means

$$F_3(x)=\begin{cases} f_3(x), & -1\le x\le 0,\\ 2\tau_1(x)-f_3(-x), & 0\le x\le 1.\end{cases}$$

Now let's write a solution to problem (36) that satisfies the first two conditions of this problem :

$$u_{22}(x,y)=\frac{1}{2}\int_{x-y}^{x+y}F_4(t)dt\,. \quad (40)$$

Substituting (40) into the third condition of problem (36) , we obtain

$$F_4(y)=-f_4(-y),\ 0\le y\le 1. \quad (41)$$

Means

$$F_4(x)=\begin{cases} f_4(x), & -1\le x\le 0,\\ -f_4(-x), & 0\le x\le 1.\end{cases}$$

Finally , let's write a solution to problem (37) that satisfies the first two conditions of this problem :

$$u_{23}(x,y)=-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_2(\xi+\eta)d\xi\,. \quad (42)$$

Substituting (42) into the third condition of problem (37) , we obtain the relation

$$\int_0^y e^{-c\eta}\Omega_2(2\eta-y)d\eta=-\Omega_2(y)\int_0^y e^{-c\eta}d\eta\,. \quad (4\,3)$$

Now substituting (38), (40) and (42) into (34), we obtain

$$u_2(x,y)=\frac{1}{2}[F_3(x+y)+F_3(x-y)]+\frac{1}{2}\int_{x-y}^{x+y}F_4(t)dt-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_2(\xi+\eta)d\xi\,. \quad (44)$$

Differentiating (44) with respect to x, we have

$$u_{2x}(x,y)=\frac{1}{2}[F_3'(x+y)+F_3'(x-y)]+\frac{1}{2}[F_4(x+y)-F_4(x-y)]-$$

$$-\frac{1}{2}\int_0^y e^{-c\eta}[\Omega_2(x+y)-\Omega_2(x-y+2\eta)]d\eta\,. \quad (45)$$

Passing in (45) to the limit at $x\to 0$ due to (39), (41) and (4 3) , we arrive at the relation

$$\nu_1(y)=\frac{1}{2}[F_3'(y)+f_3'(-y)]+\frac{1}{2}[F_4(y)-f_4(-y)]-$$

$$-\frac{1}{2}\Omega_2(y)\int_0^y e^{-c\eta}d\eta+\frac{1}{2}\int_0^y e^{-c\eta}\Omega_2(2\eta-y)d\eta=$$

$$=\frac{1}{2}[2\tau_1'(y)+f_3'(-y)+f_3'(-y)]+\frac{1}{2}[-f_4(-y)-f_4(-y)]+\int_0^y e^{-c\eta}\Omega_2(2\eta-y)d\eta\,,$$

those.

$$\nu_1(y)=\tau_1'(y)+f_3'(-y)-f_4(-y)+\int_0^y e^{-c\eta}\Omega_2(2\eta-y)d\eta,\ 0\le y\le 1$$

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$$\int_0^y e^{-c\eta}\Omega_2(2\eta - y)d\eta = \nu_1(y) - \tau_1'(y) - f_3'(-y) + f_4(-y).$$

In the integral, which is on the left side of this equality, by replacing $2\eta - y$ with $-z$, we get

$$\int_{-y}^{y} e^{-\frac{c}{2}(y-z)}\Omega_2(-z)dz = 2[\nu_1(y) - \tau_1'(y) - f_3'(-y) + f_4(-y)]. \quad (4\ 6\ )$$

Differentiating (4 6) and taking into account again this equality (4 6), after some calculations, we find

$$\Omega_2(y) = -\omega_2(-y)e^{cy} - c[\nu_1(y) - \tau_1'(y) - f_3'(-y) + f_4(-y)]e^{cy} +$$
$$+ 2[\nu_1'(y) - \tau_1''(y) + f_3''(-y) - f_4'(-y)]e^{cy}. \quad (47)$$

Now passing in equation (16) to the limit at $y \to 0$, we find :

$$\omega_{11}(x) = f_1''(x) - f_2(x),\ 0 \le x \le 1,$$

the notation is introduced here

$$\omega_1(x+y) = \begin{cases} \varpi_{11}(x+y),\ 0 \le x+y \le 1, \\ \varpi_{12}(x+y),\ 1 \le x+y \le 2. \end{cases}$$

Further, passing in equations (17) ($i=2$) and (16) to the limit at $x \to 0$, due to (1 0) and (12), we obtain the relations

$$\mu_1(y) - \tau_1''(y) = \Omega_2(y)e^{-cy},\ 0 \le y \le 1,\ \ \mu_1(y) - \tau_1'(y) = \omega_{11}(y)e^{-cy},\ 0 \le y \le 1.$$

Eliminating from these relations the function $\mu_1(y)$, we find

$$\Omega_2(y) = \omega_{11}(y) - [\tau_1''(y) - \tau_1'(y)]e^{cy},\ 0 \le y \le 1. \quad (48)$$

If we substitute (48) into (47), then after some calculations we arrive at an ordinary differential equation for $\nu_1(y)$:

$$\nu_1'(y) - \frac{c}{2}\nu_1(y) = \frac{1}{2}e^{cy}\tau_1''(y) + \frac{1-c}{2}e^{cy}\tau_1'(y) + s_2(y),\ 0 \le y \le 1, \quad (49)$$

where

$$s_2(y) = \frac{1}{2}\omega_{11}(y) + \frac{1}{2}e^{cy}\omega_2(-y) - \frac{c}{2}[f_3'(-y) - f_4(-y)]e^{cy} - [f_3''(-y) - f_4'(-y)]e^{cy}.$$

Solving equation (49) under the condition $\nu_1(0) = f_1'(0)$, we get the ratio

$$\nu_1(y) = \frac{1}{2}e^{cy}\tau_1'(y) + \frac{2-3c}{4}\int_0^y e^{\frac{c}{2}(y+z)}\tau_1'(z)dz + \beta_2(y),\ 0 \le y \le 1, \quad \text{(fifty)}$$

here

$$\beta_2(y) = -\frac{1}{2}e^{\frac{c}{2}y}f_2(0) + e^{\frac{c}{2}y}f_1'(0) + \int_0^y e^{\frac{c}{2}(y-z)}s_2(z)dz.$$

Now the task $M^1_{1(-1)c}$ consider in the area G_1. Passing in equations (16) and (17) ($i=3$) to the limit at $x \to 1$, due to (13) and (15) we get

$$\mu_2(y) - \tau_2'(y) = \omega_{12}(1+y)e^{-cy},\ \ \mu_2(y) - \tau_2''(y) = \omega_2(1+y)e^{-cy},\ 0 \le y \le 1.$$

Excluding from these equations the function $\mu_2(y)$, we find

$$\omega_{12}(1+y) = \omega_3(1+y) + [\tau_2''(y) - \tau_2'(y)]e^{cy}. \quad (51)$$

Further, we write the solution of equation (16) , satisfying conditions (2), (10), (13):

$$u_1(x,y)=\int_0^y \tau_1(\eta)G_\xi(x,y;0,\eta)d\eta+\int_0^y \tau_2(\eta)G_\xi(x,y;1,\eta)d\eta+\int_0^1 f_1(\xi)G(x,y;\xi,0)d\xi-$$

$$-\int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)G(x,y;\xi,\eta)d\xi-\int_0^y e^{-c\eta}d\eta\int_{1-\eta}^1\omega_{12}(\xi+\eta)G(x,y;\xi,\eta)d\xi\,. \quad (52)$$

Differentiating (52) with respect to x, we get

$$u_{1x}(x,y)=-\int_0^y \tau_1'(\eta)N(x,y;0,\eta)d\eta+\int_0^y \tau_2'(\eta)N(x,y;1,\eta)d\eta+\int_0^1 f_1'(\xi)N(x,y;\xi,0)d\xi+$$

$$+\int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)N_\xi(x,y;\xi,\eta)d\xi+\int_0^y e^{-c\eta}d\eta\int_{1-\eta}^1\omega_{12}(\xi+\eta)N_\xi(x,y;\xi,\eta)d\xi\,.$$

In the last integral of this equality, replacing $\xi+\eta$ by $1+z$ and substituting (51) into the last equality , after some calculations and transformations, we get

$$u_{1x}(x,y)=-\int_0^y \tau_1'(\eta)N(x,y;0,\eta)d\eta+\int_0^y \tau_2'(\eta)N(x,y;1,\eta)d\eta+\int_0^1 f_1'(\xi)N(x,y;\xi,0)d\xi+$$

$$+\int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)N_\xi(x,y;\xi,\eta)d\xi+\int_0^y\omega_3(1+z)dz\int_z^y e^{-c\eta}N_\xi(x,y;1-\eta+z,\eta)d\eta-$$

$$-f_2(0)\int_0^y e^{-c\eta}N_\xi(x,y;1-\eta,\eta)d\eta-(c+1)\int_0^y \tau_2'(\eta)d\eta\int_\eta^y e^{c(\eta-z)}N_\xi(x,y;z-\eta,z)dz+$$

$$+\int_0^y \tau_2'(\eta)d\eta\int_\eta^y e^{c(\eta-z)}N_z(x,y;1-z+\eta,z)dz\,. \quad (53)$$

Passing in (53) to the limit at $x\to 0$, due to (50) , we get

$$\frac{1}{2}e^{cy}\tau_1'(y)+\frac{2-3c}{4}\int_0^y e^{\frac{c}{2}(y+\eta)}\tau_1'(\eta)d\eta+\beta_2(y)=$$

$$=-\int_0^y \tau_1'(\eta)N(0,y;0,\eta)d\eta+\int_0^y \tau_2'(\eta)N(0,y;1,\eta)d\eta+\int_0^1 f_1'(\xi)N(0,y;\xi,0)d\xi+$$

$$+\int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)N_\xi(0,y;\xi,\eta)d\xi-f_2(0)\int_0^y e^{-c\eta}N_\xi(0,y;1-\eta,\eta)d\eta-$$

$$-(c+1)\int_0^y \tau_2'(\eta)d\eta\int_\eta^y e^{c(\eta-z)}N_\xi(0,y;1-z+\eta,z)dz+\int_0^y \tau_2'(\eta)d\eta\int_\eta^y e^{c(\eta-z)}N_z(0,y;1-z+\eta,z)dz+$$

$$+\int_0^y\omega_3(1+z)dz\int_z^y e^{-c\eta}N_\xi(0,y;1-\eta+z,\eta)d\eta\,.$$

If in this equality we introduce the notation

$$K_1(y,\eta)=\frac{2-3c}{2}e^{\frac{c}{2}(\eta-z)}+2e^{-cy}N(0,y;0,\eta),\; K_2(y,\eta)=-2e^{-cy}N(0,y;1,\eta)+$$

$$+2(c+1)\int_\eta^y e^{c(\eta-y-z)}N_\xi(0,y;1-z+\eta,z)dz-2\int_\eta^y e^{c(\eta-y-z)}N_z(0,y;1-z+\eta,z)dz\,,$$

$$g_1(y) = -2e^{-cy}\beta_2(y) + 2e^{-cy}\int_0^1 f_1'(\xi)N(0,y;\xi,0)d\xi + 2\int_0^y e^{-c(y+\eta)}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)N_\xi(0,y;\xi,\eta)d\xi -$$

$$-2f_2(0)\int_0^y e^{-c(y+\eta)}N_\xi(0,y;1-\eta,\eta)d\eta + 2\int_0^y \omega_3(1+z)dz\int_z^y e^{-c(y+\eta)}N_\xi(0,y;1-\eta+z,\eta)d\eta,$$

then we get

$$\tau_1'(y) + \int_0^y K_1(y,\eta)\tau_1'(\eta)d\eta + \int_0^y K_2(y,\eta)\tau_2'(\eta)d\eta = g_1(y). \quad (54)$$

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Now passing into (53) to the limit at $x \to 1$, due to (32), after lengthy calculations and transformations, we obtain

$$\tau_2'(y) + \int_0^y K_4(y,\eta)\tau_2'(\eta)d\eta + \int_0^y K_3(y,\eta)\tau_1'(\eta)d\eta = g_2(y), \quad (5)$$

where

$$K_3(y,\eta) = -N(1,y;0,\eta),\ K_4(y,\eta) = -N(1,y;1,\eta) -$$

$$-(c+1)\int_\eta^y e^{c(\eta-z)}N_\xi(1,y;1-z+\eta,z)dz + \int_\eta^y e^{c(\eta-z)}N_z(1,y;1-z+\eta,z)dz,$$

$$g_1(y) = \beta_1(y) - \int_0^1 f_1'(\xi)N(1,y;\xi,0)d\xi - \int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)N_\xi(1,y;\xi,\eta)d\xi -$$

$$-\int_0^y \omega_3(1+z)dz\int_z^y e^{-c\eta}N_\xi(1,y;1-\eta+z,\eta)d\eta + f_2(0)\int_0^y N_\xi(1,y;1-\eta,\eta)d\eta.$$

Solving system (54), (55) , we find the functions $\tau_1'(y)$ and $\tau_2'(y)$ and thus , the solution of the problem $M^1_{1(-1)c}$ is unique.

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