GALAXY INTERNATIONAL INTERDISCIPLINARY RESEARCH JOURNAL (GIIRJ)
ISSN (E): 2347-6915
Vol. 10, Issue 11, Nov. (2022)

DIFFERENT FUNCTION AND REGULARIZATION FOR POLYHARMONIC FUNCTIONS FOR SOME REGIONS IN .R^m

Sotimboeva Zarifakhon
Senior Lecturer of the Department of Higher
Mathematics at Namangan State University

ANNOTATION

In this work, some properties and regulation of the Carlemann function are studied to determine the integral formula of n-th order polygarmonic functions (D ^ n u (y) = 0) and their properties that satisfy the condition 2n≥m in certain unbounded areas of real m-dimensional Euclidean space.

Keywords: real m-dimensional Euclidean, region lying in a layer.

Based on these studies, M.M. Lavrentyev introduced an important concept- the Carleman function and with its help built a regularization of the problem. With the help of the method of M.M. Lavrentyev, Sh. Yarmukhamedov obtained the regularization and solvability of the Cauchy problem for the Laplace equation in bounded areas [3]. In 2009, Juraeva N (second author) of the article obtained the regularization and solvability of the Cauchy problem for polyharmonic equations of order in some unlimited regions (with arbitrary odd odd regions). and even when) [4] - [8]. $nmm\ \ 2n < m$

Let's assume that the solution of the problem (1) - (2) exists and is continuously differentiable, up to the end points of the boundary and satisfies a certain growth condition (correctness class), which ensures the uniqueness of the solution.$u(y)2n-1$

Having solved the problem (1),(2) with the help of its solution we obtain theorems of the Fragman-Lindelof type. Fragman-Lindelof type theorems were the subject of research on the works of M.A. Evgrafov, I.A. Chegis and A.F. Lavrentyev,I.S. Arshon and others.

In 1960, M.A. Evgrafov and I.A. Chegis in the article - Generalization of the Fragman-Lindelof type theorem for analytic functions to harmonic functions in space - (DAN USSR, volume 134, number 2, 252-262) proved

Theorem 1. Let be the harmonic function in the cylinder , , . If the conditions are met $u(r,\phi,x)0 \le r \le \mathrm{a}0 \le \phi < 2\pi - \infty < \mathrm{x} < \infty$

$$u(\mathrm{a},\phi,\mathrm{x}) = 0, \left|\frac{\partial u}{\partial r}(a,\varphi,x)\right| < c, \quad \max_{(r,\phi)} |u(r,\phi,x)| < c\, exp\, e^{\frac{\pi|x|}{2(a+\varepsilon)}}, \varepsilon > 0$$

then $u(r,\phi,\mathrm{x}) \equiv 0$

Theorem 2. Let the harmonic function in the cone , ,. If the conditions are met $u(r,\theta,\phi)0 < r < \infty 0 \le \phi < 2\pi 0 \le \theta \le \theta_0 < \pi$

$$u(r,\theta_0,\phi) = 0, \left|\frac{\partial u}{\partial \theta}(r,\theta_0,\varphi)\right| < c, \quad \max_{(\theta,\phi)} |u(r,\theta,\phi)| < c\, exp(r + \frac{1}{r})^{\frac{\pi}{2\theta_0} - \varepsilon}, \varepsilon > 0$$

then $u(r,\theta,\phi) \equiv 0$

In 1961, I.A. Chegis in the article - The theorem of the Fragman-Lindelof type for harmonic functions in a rectangular cylinder - (Dokl. AS USSR, volume 136, number 3, 556-9) proved

Theorem 1. - Harmonic function in a cylinder over a rectangle , ; . If the conditions are met $u(x,y,t) 0 \le x \le a 0 \le y \le b - \infty \le t \le \infty$

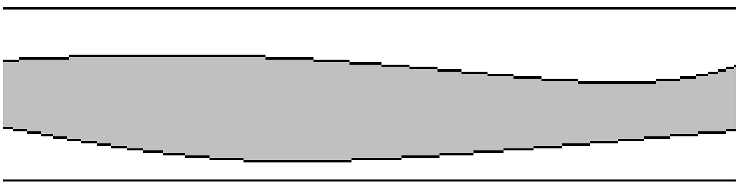
$u(x,y,t)|_{\Gamma} = 0$, where is the G-surface of the cylinder;

$$\left|\frac{\partial u(x,0,t)}{\partial y}\right| < c, \left|\frac{\partial u(x,b,t)}{\partial y}\right| < c \ , \ \max_{(x,y)} u(x,y,t) < c\, exp\, e^{\pi|t|/b+\varepsilon}, \varepsilon > 0$$

then. $u(x,y,t) \equiv 0$

It should be noted that the theorem uses the condition of bounding the normal derivativeof tolk 'o on two opposite faces of a rectangular cylinder. For an infinite layer, i.e., a region of view, the statement of the theorem remains valid. $-\infty < x < \infty, 0 \le y \le b, -\infty < t < \infty$

E.M. Landis in the book -Equations of the second order of elliptical and parabolic types. Moscow, 1971 g.55 p.) - set the problem in the form - Let the cylinder contain an area that goes to infinity (in one or both directions - all the same) in the boundary of G of this region as smooth as you like $0 \le \sum_{k=0}^{n-1} x_i^2 < 1$



Let the solution and equation be defined in the region as smooth as possible up to the boundary and . Does it follow from this that unlimited (exponentially grows when going to infinity).)

$$\Delta\Delta u = 0 u|_{\Gamma} = 0, \quad \left.\frac{\partial u}{\partial n}\right|_{\Gamma} = 0$$

Defining the functions and the following equals: $\phi_\sigma(y,x)$ $\Phi_\sigma(y,x)(s > 0, \sigma \ge 0, m -$ размерность пространство)

if $m = 2k+1, k = 2,3, \ldots$

$$\phi_\sigma(y,x) = c_1 \frac{\partial^{k-1}}{\partial s^{k-1}} \int_0^\infty Im \frac{\varsigma(i\sqrt{s+u^2}+y_m)}{i\sqrt{s+u^2}+y_m-x_m} \frac{du}{\sqrt{u^2+s}}, \qquad (3)$$

if then $m = 2k, \quad k = 2,3, \ldots$

$$\phi_\sigma(y,x) = c_1 \frac{\partial^{k-2}}{\partial s^{k-2}} Im\left(\frac{\zeta(i\sqrt{s}+y_m)}{\sqrt{s}(i\sqrt{s}+y_m-x_m)}, \right) \qquad (4)$$

Where is $\varsigma(\omega) = \quad exp\left(\sigma\omega^2 - achi\rho_1(\omega - h/2) - bchi\rho_0(\omega - h/2)\right)$,

$$c_1 = c_0\, exp\left(-\sigma x_m^2 + achi\rho_1(x_m - \frac{h}{2}) + bchi\rho_0\left(x_m - \frac{h}{2}\right)\right), \quad b > b_0\left(cos\frac{\rho_0 h}{2}\right)^{-1}$$

For all odd , as well as even with the condition we assume $m \ge 3 m 2n < m$

$$\Phi_\sigma(y,x) = C_{n,m} r^{2(n-1)} \varphi_\sigma(y,x) \quad , C_{n,m} = (-1)^{\frac{m}{2}-1}\left(\Gamma(n - \frac{m}{2} + 1) 2^{2n-1} \pi^{\frac{m}{2}} \Gamma(n)\right)^{-1} \qquad (5)$$

And for even with the condition defining the function at , : $m 2n \ge m \Phi_\sigma(y,x) s > 0, \sigma \ge 0, a \ge 0 2n \ge m$

$$\Phi_\sigma(y,x) = C_{n,m} \int_{\sqrt{s}}^\infty Im\left[\frac{exp(\sigma w + w^2) - achi\rho_1\left(w - \frac{h}{2}\right)}{\omega - x_1}\right] (u^2 - s)^{n-k} du, \ \omega = iu + y_1 \qquad (6)$$

$$C_{n,m} = (-1)^{\frac{m}{2}-1}\left(\Gamma(n - \frac{m}{2} + 1) 2^{2n-1} \pi^{\frac{m}{2}} \Gamma(n)\right)^{-1} \qquad \text{Get}$$

Theorem 1. For the function is the place $\Phi_\sigma(y,x)$

$$\Phi_\sigma(y,x) = \begin{cases} C_{n,m} r^{2n-m} \ln r + G_\sigma(y,x), & 2n \geq m, m - \text{чётное число}, \\ C_{n,m} r^{2n-m} + G_\sigma(y,x), & \text{в остальных случаях}, \end{cases}$$

Where is

$$C_{n,m} = (-1)^{\frac{m}{2}-1} \left(\Gamma(n - \frac{m}{2} + 1) 2^{2n-1} \pi^{\frac{m}{2}} \Gamma(n) \right)^{-1}$$

where regular by variable y and continuously differentiated by $.G_\sigma(y,x) D \cup \partial D = \overline{D}$

Theorem 2. The function , defined by formula (3) is a polyharmonic function of order n po at $\Phi_\sigma(y,x) y s > 0$.

Theorem 3. With a fixed function, satisfies$x \in D \Phi_\sigma(y,x)$

$$\sum_{k=0}^{n-1} \int_{\partial D \setminus S} \left[|\Delta^k \Phi_\sigma(y,x)| - \left| \frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial \bar{n}} \right| \right] ds_y \leq C(x) \varepsilon(\sigma),$$

where the constant depends on $C(x)$x and the -external normal k when $.\bar{n} \partial D \varepsilon(\sigma) \to 0 \sigma \to \infty$

Consequence 3. The function , defined by formula (3) is a Carlemann function for the point and part of $.\Phi_\sigma(y,x) x \in D \partial D \setminus S$

Theorem 4. Let the solution of problems and , having continuous partial derivatives of order up to the end points of the boundary. If the growth condition is met for anyone $u(x)(1)-(2)2n-1 \partial D y \in D$

$$\sum_{k=0}^{n-1} |\Delta^k u(y)| + |grad \Delta^{n-k-1} u(y)| \leq c_0 \exp(\exp(\rho_1 |y'|))$$

and for any fulfilled the condition of growth$y \in \partial D$

$\sum_{k=0}^{n-1} |\Delta^k u(y)| + \left| \frac{\partial}{\partial n} \Delta^{n-k-1} u(y) \right| \leq c_0 \exp\left(a \cos \rho_2 \left(y_1 - \frac{h}{2} \right) \exp(\rho_2 |y'|) \right)$ Where is $\rho_1 < \rho_2 < \rho_3 < \rho$

Then for anyone the integral representation is true.$x \in D$

$$u(x) = \sum_{k=0}^{n-1} \int_{\partial D} \left[\Delta^k \Phi_\sigma(y,x) \frac{\partial \Delta^{n-k-1} u(y)}{\partial n} - \Delta^{n-k-1} u(y) \frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial n} \right] ds. \quad x \in D$$

Theorem 5. Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any $u(x)(1)-(2)2n-1 \partial D y \in D$

$$\sum_{k=0}^{n-1} |\Delta^k u(y)| + |grad \Delta^{n-k-1} u(y)| \leq c_0 \exp(\exp(\rho_1 |y'|))$$

and if the growth condition is met $\forall y \in \partial D$

$$\sum_{k=0}^{n-1}\int_{\partial B}\Delta^k u\cdot\exp\left(-\rho_2|y'|\right)ds<\infty,$$
$$\sum_{k=0}^{n-1}\frac{\partial\Delta^k u}{\partial n}\cdot\exp\left(-\rho_2|y'|\right)ds<\infty$$
where $\rho_2<\rho_1<\rho$.

Then the integral representation is true. $\forall x\in D$

$$u(x)=\sum_{k=0}^{n-1}\int_{\partial D}\left[\Delta^k\Phi_\sigma(y,x)\frac{\partial\Delta^{n-k-1}u(y)}{\partial n}-\Delta^{n-k-1}u(y)\frac{\partial\Delta^k\Phi_\sigma(y,x)}{\partial n}\right]ds.$$

Theorem 6. Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any $u(x)(1)-(2)2n-1\partial Dy\in D$

$$\sum_{k=0}^{n-1}|\Delta^k u(y)|+|grad\Delta^{n-k-1}u(y)|\le c_0\,exp(exp(\rho_1|y'|))$$

And if the growth condition is met $\forall y\in\partial D$

$$\Delta^k u=0,\quad \forall k\in[0,n-1],\quad \forall y\in\partial D,$$
$$\sum_{k=0}^{n-1}\frac{\partial\Delta^k u}{\partial n}\cdot\exp\left(-\rho_2|y'|\right)ds<\infty,\quad \forall y\in\partial D,$$
where $\rho_2<\rho_1<\rho$.

then the integral representation is fair $\forall x\in D$

$$u(x)=\sum_{k=0}^{n-1}\int_{\partial D}\Delta^{n-k-1}\Phi_\sigma(y,x)\frac{\partial\Delta^k u(y)}{\partial n}ds.$$

Consequence. Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any $u(x)(1)-(2)2n-1\partial Dy\in D$

$$\sum_{k=0}^{n-1}|\Delta^k u(y)|+|grad\Delta^{n-k-1}u(y)|\le c_0\,exp(exp(\rho_1|y'|))$$

If the growth condition is met $\forall y\in\partial D$

$\sum_{k=0}^{n-1}|\Delta^k u(y)|+\left|\frac{\partial\Delta^{n-k-1}u(y)}{\partial y}\right|\le c_0$ where $\rho_2<\rho_1<\rho$.

Тогда справедливо in $\forall x\in D$ (x)=0

Consequence. Let the solution of the problem , having continuous partial derivatives of order up to the end points of the boundary . If the growth condition is met for any$u(x)(1)-(2)2n-1\partial Dy\in D$

$$\sum_{k=0}^{n-1}|\Delta^k u(y)|+|grad\Delta^{n-k-1}u(y)|\le c_0\,exp(exp(\rho_1|y'|))$$

If the growth condition is met $\forall y\in\partial D$

$$\Delta^k u=0,\quad \forall k\in[0,n-1],\quad \forall y\in\partial D,$$
$$\sum_{k=0}^{n-1}\frac{\partial\Delta^k u}{\partial n}\cdot\exp\left(-\rho_2|y'|\right)ds<\infty,\quad \forall y\in\partial D,$$
Where is. $\rho_2<\rho_1<\rho$

Then u(x)=0 is true $\forall x \in D$
These results are in a sense a response to the problems posed by E.M. Landis for polyharmonic functions of order . n

Theorem 7. Let be a solution to the problem having continuous partial derivatives of order up to the endpoints of the boundary . If the growth condition is met for anyone $u(x)(1)-(2)2n-1\partial D y \in D$

$$\sum_{k=0}^{n-1} |\Delta^k u(y)| + |grad\Delta^{n-k-1} u(y) \le c_0 \, exp(exp(\rho_1|y'|))|$$

and

$$\forall y \in \partial D \backslash S \left|\frac{\partial \Delta^{n-1-k} u(y)}{\partial \bar{n}}\right| + |\Delta^{n-1-k} u(y)| \le 1 \qquad ,$$

then fair

$$|u(x) - u_\sigma(x)| \le MC(\sigma)\, exp(-\sigma x_m)\,, \quad \sigma \ge \sigma_0 > 0, \quad x \in D.$$

Where is $u_\sigma(x) = \sum_{k=0}^{n-1} \int_S \left[G_{n-k-1}(y)\Delta^k \Phi_\sigma(y,x) - F_{n-k-1}(y) \frac{\partial \Delta^k \Phi_\sigma(y,x)}{\partial \bar{n}}\right] ds$

$C(\sigma)$- A polygamy with well-defined coefficients.

REFERENCES

1. Sobolev S.L. Introduction to the theory of cubature*formulas.* M. Nauka 1974. ct 514-673.
2. Lavrentyev M.M.. About some incorrect problems of mathematical physics. Novosibirsk, 1962, p. 243.
3. Sh. Yarmukhamedov. Cauchy's problem for a polyharmonic equation. Reports of the Russian Academy of Sciences 2003 Volume 388 Of Articles 162-165.