GALAXY INTERNATIONAL INTERDISCIPLINARY RESEARCH JOURNAL (GIIRJ)
ISSN (E): 2347-6915
Vol. 10, Issue 11, Nov. (2022)

METHODS OF SOLVING NON-STANDARD MATHEMATICAL PROBLEMS

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ABSTRACT

This article presents convenient ways to solve some non-standard problems that appear in science olympiad problems for mathematics.

Keywords: Olympiad, education, problem, equation, pedagogy, quality of education, educational methods.

NOSTANDART MATEMATIK MASALALARNI YECHISH USULLARI

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ANNOTATSIYA

Ushbu maqolada matematika fanidan uchun fan olimpiada masalalarida uchraydigan ba'zi bir nostandart masalalarni yechishning qulay usullari keltirilgan.

Kalit so'zlar: olimpiada, ta'lim, masala, tenglama, pedagogika, ta'lim sifati, ta'lim metodlari.

Yurtimiz azaldan matematika faniga katta hissa qo'shgan olimlari bilan butun dunyoga tanilgan. Bularga misol sifatida ulug' boblarimiz Muhammad ibn Muso al-Xorazmiy, Ahmad Farg'oniy, Abu Rayhon Beruniy, Ibn-Sino, Forobiy, Ulug'bek kabi bobokalonlarimizni ko'rsatish mumkin. Ularning izdoshlari sifatida O'zbekistonda zamonaviy matematika muammolari bilan shug'ullanuvchi maktab asoschilari bo'lmish akademiklar T.N. Qori-Niyoziy, T.A. Sarimsoqov, S.X. Sirojiddinov, T.J. Jo'rayev , T.A. Azlarov, N.Yu. Satimov kabi juda ko'p ustozlarimizni ko'rsatish mumkin. O'quvchilar matematika fani bo'yicha olimpiadalarda muvaffaqiyatli qatnashishlari uchun juda ko'p qo'shimcha adabiyotlarni o'rganishlari, nostandart masala-misollarni yechishni mashq qilishlari talab etiladi. Bu ularning tug'ma qobiliyatlarini kuchayishga, masalalarni yechishda "olimpiadacha mushohida" qilish tajribasini oshishiga olib keladi. Biz quyidagicha nostandart masalalarni keltiramiz va yechimini ko'rsatamiz.

1-Misol. Ushbu tenglamani qanoatlantiruvchi barcha (x, y) butun sonlar juftliklarini toping:

$$4x^3+4x^2y-15xy^2-18y^3-12x^2+6xy+36y^2+5x-10y=0.$$

Yechim: Berilgan tenglamaning chap tomonidagi ko'phadni $P(x,y)$ deb belgilaymiz va uni x noma'lumga nisbatan quyidagi ko'phad ko'rinishiga keltiramiz:

$$P(x,y)=4x^3+4(y-3)x^2-(15y^2-6y-5)x-2y(9y^2-18y+5)=0 \qquad (1)$$

Bunda, y butun son bo'lganda ($y\in Z$), (1) x ga nisbatan butun koeffisientli algebraik tenglama bo'ladi va uning butun x ildizi $2y(9y^2-18y+5)$ osod hadning bo'luvchisidan iborat bo'ladi. Demak, (1) tenglamaning butun (x,y) yechimini $x=2y$, $y\in Z$, ko'rinishda izlash mumkin:

$$P(2y,y)=4\cdot(2y)^3+4(y-3)(2y)^2-(15y^2-6y-5)\cdot 2y-2y(9y^2-18y+5)=0\Rightarrow$$
$$\Rightarrow 2y[16y^2+8y(y-3)-(24y^2-24y)]=0\Rightarrow 2y\cdot 0=0\Rightarrow 0\equiv 0.$$

Bu yerdan kelib chiqadiki har qanday $y\in Z$ uchun $(x,y)=(2y,y)$ juftliklar berilgan tenglamaning butun yechimlari bo'ladi. Bundan $P(x,y)$ ko'phad $x-2y$ ikkihadga qoldiqsiz bo'linishi kelib chiqadi. Bo'linmani topish uchun $P(x,y)$ ko'phaddan $x-2y$ ko'paytuvchini ajratib olishga harakat qilamiz:

$$P(x,y)=4x^2(x-2y)+12xy(x-2y)+9y^2(x-2y)-12x(x-2y)-$$
$$-18y(x-2y)+5(x-2y)=(x-2y)[4x^2+12xy+9y^2-12x-18y+5]=0\ . \qquad (2)$$

Yuqoridagi (1) tenglamadan uning butun ildizlarini $x=y$ ko'rinishda ham izlash mumkinligi kelib chiqadi. Bu holda (2) tenglamadan quyidagi natija kelib chiqadi:

$$(x-2x)[4x^2+12x+9x^2-12x-18x+5]=0\Rightarrow\left[\begin{array}{c}x=0\\ 25x^2-30x+5=0\end{array}\right.\Rightarrow\left[\begin{array}{c}x=0\\ x=1,\ x=1/5\end{array}\right..$$

Demak, (0,0) va (1,1) juftliklar ham berilgan tenglamaning butun yechimlari bo'ladi. Bunda $y=1$ (2) tenglamaning ildizi bo'lgani uchun undagi kvadrat qavs ichidagi ifodada $y-1$ ko'paytuvchini hosil qilishga harakat qilamiz:

$$4x^2+12xy+9y^2-12x-18y+5=4x^2+12x(y-1)+9(y^2-2y+1)-4=$$
$$=[2x+3(y-1)]^2-2^2=[2x+3(y-1)+2]\cdot[2x+3(y-1)-2]=(2x+3y-1)(2x+3y-5).$$

Natijada berilgan tenglama quyidagi tenglamalar birlashmasiga keladi:

$$(x-2y)(2x+3y-1)(2x+3y-5)=0\Rightarrow\left[\begin{array}{l}x-2y=0\\ 2x+3y-1=0\\ 2x+3y-5=0\end{array}\right.. \qquad (3)$$

(3) tenglamalar birlashmasining I tenglamasi yechimi $(2k,\ k)$, $k\in Z$, ko'rinishda ekanligini yuqorida ko'rgan edik.

II tenglamani yechib, quiydagi javobga kelamiz:

$$2x+3y-1=0\Rightarrow y\in Z,\ x=\frac{1-3y}{2}=\frac{1-y}{2}-y\in Z\Rightarrow y=2k+1,\ x=-3k-1\ (k\in Z).$$ III tenglamani yechib, quiydagi javobga kelamiz:

$$2x+3y-5=0\Rightarrow y\in Z,\ x=\frac{5-3y}{2}=2-y+\frac{1-y}{2}\in Z\Rightarrow y=2k+1,\ x=-3k+1\ (k\in Z).$$ Bu yerdan ko'rinadiki, II va III tenglamalar yechimlarini birgalikda

$$x=-3k\pm 1,\ y=2k+1,\ k\in Z$$

ko'rinishda yozish mumkin.

Demak, berilgan tenglamaning barcha butun $(x,\ y)$ yechimlari juftliklari $(2k,\ k)$ va $(-3k\pm 1,\ 2k+1)$, $k\in Z$, ko'rinishda bo'ladi.

2-Misol. $P(x^2)+x[3P(x)+P(-x)]=[P(x)]^2+2x^2,\quad \forall x\in R,$ tenglikni qanoatlantiruvchi barcha haqiqiy koeffisientli $P(x)$ ko'phadlarni toping.

Yechim: Masala shartidagi tenglik ixtiyoriy x uchun o'rinli bo'lgani uchun x o'rniga $-x$ qo'yishimiz mumkin:

$$\begin{cases} P(x^2)+x[3P(x)+P(-x)]=[P(x)]^2+2x^2 \\ P(x^2)-x[3P(-x)+P(x)]=[P(-x)]^2+2x^2 \end{cases}.$$

Bu sistemadagi tengliklarni hadma-had ayiramiz:

$$4x[P(x)+P(-x)]=[P(x)]^2-[P(-x)]^2 \Rightarrow \left[\begin{array}{l} P(x)+P(-x)=0 \\ P(x)-P(-x)=4x \end{array}\right..$$

Birinchi $P(x)+P(-x)=0$ holda $P(x)$ ko'phad toq funksiya va shu sababli

$$P(x)=p_0x+p_1x^3+p_2x^5+\cdots+p_nx^{2n+1}=\sum_{k=0}^{n}p_kx^{2k+1},\ p_n\neq 0 \qquad (1)$$

ko'rinishda bo'ladi. Bu holda berilgan tenglamadan quyidagi natijalarni olamiz:

$$P(x^2)-x^2=P^2(x)-2xP(x)+x^2 \Rightarrow P(x^2)-x^2=[P(x)-x]^2 \Rightarrow$$

$$\Rightarrow (p_0-1)x^2+\sum_{k=1}^{n}p_kx^{2(2k+1)}=[(p_0-1)x+\sum_{k=1}^{n}p_kx^{2k+1}]^2=$$

$$=(p_0-1)^2x^2+2(p_0-1)x\sum_{k=1}^{n}p_kx^{2k+1}+[\sum_{k=1}^{n}p_kx^{2k+1}]^2 \Rightarrow$$

$$\Rightarrow (p_0-1)+\sum_{k=1}^{n}p_kx^{4k}=(p_0-1)^2+2(p_0-1)\sum_{k=1}^{n}p_kx^{2k}+[\sum_{k=1}^{n}p_kx^{2k}]^2 \qquad (2)$$

Bu tenglikning ikkala tomonidagi ozod hadlalrni taqqoslab,

$p_0-1=(p_0-1)^2 \Rightarrow p_0=1$ yoki $p_0=2$ ekanligini ko'ramiz.

Dastlab p_0=1 holni qaraymiz. Bu holda (2) tenglik quyidagi ko'rinishga keladi:

$$\sum_{k=1}^{n}p_kx^{4k}=[\sum_{k=1}^{n}p_kx^{2k}]^2=\sum_{k,j=1}^{n}p_kp_jx^{2(k+j)} \qquad (3)$$

Bu tenglikni ikkala tomonidagi x darajalari oldidagi koeffisientlarni tenglashtirib, noma'lum p_k, k=1,2,..., n , koeffisientlar uchun quyidagi tenglamalarga ega bo'lamiz:

$x^{4n}: p_n^2=p_n\ (p_n\neq 0)\Rightarrow p_n=1;\ x^{4n-2}:\quad 2p_{n-1}p_n=0\Rightarrow p_{n-1}=0;$

$x^{4n-4}: 2p_{n-2}p_n+p_{n-1}^2=p_{n-1}\Rightarrow p_{n-2}=0;$

$x^{4n-6}:\quad 2(p_{n-3}p_n+p_{n-2}p_{n-1})=0\Rightarrow p_{n-3}=0;...$

Bu jarayonni davom ettirib, p_k=0, k=1,2,..., $n-1$, ekanligini topamiz.

Demak, p_0=1 holda (1) ko'phad $P(x)=x+x^{2n+1}$ (*) ko'rinishda bo'ladi.

Endi p_0=2 holni qaraymiz. Bu holda (2) tenglik quyidagi ko'rininshga keladi:

$$\sum_{k=1}^{n}p_kx^{4k}=2\sum_{k=1}^{n}p_kx^{2k}+[\sum_{k=1}^{n}p_kx^{2k}]^2 \qquad (4)$$

Bu tenglikning chap tomonidagi x ning eng kichik darajasi x^4, o'ng tomonda esa x^2. Bundan p_1=0 ekanligi kelib chiqadi. Bu holda (4) tenglik quyidagi ko'rinishga keladi:

$$\sum_{k=2}^{n}p_kx^{4k}=2\sum_{k=2}^{n}p_kx^{2k}+[\sum_{k=2}^{n}p_kx^{2k}]^2 \qquad (5)$$

Bu tenglikning chap tomonidagi x ning eng kichik darajasi x^8, o'ng tomonda esa x^4. Bundan $p_2=0$ ekanligi kelib chiqadi. Bu holda (5) tenglik quyidagi ko'rinishga keladi:

$$\sum_{k=3}^{n} p_k x^{4k} = 2\sum_{k=3}^{n} p_k x^{2k} + [\sum_{k=3}^{n} p_k x^{2k}]^2 \Rightarrow p_3 = 0 \qquad (6)$$

Bu jarayonni davom ettirib

$$\sum_{k=n-1}^{n} p_k x^{4k} = 2\sum_{k=n-1}^{n} p_k x^{2k} + [\sum_{k=n-1}^{n} p_k x^{2k}]^2 \Rightarrow p_{n-1} = 0;$$

$$p_n x^{4n} = 2p_n x^{2n} + p_n x^{4n} \Rightarrow p_n = 0$$

natijani olamiz. Demak, bu holda $P(x)=2x$. Bu natija (*) ko'phaddan $n=0$ holda kelib chiqadi. Tekshirish orqali $P(x)=x$ birhad ham masala yechimi bo'lishini ko'rish mumkin.

Demak, birinchi holda masala yechimi $P(x) = x + x^{2n+1}(n = 0,1,2,...), P(x) = x$ ko'rinishda bo'ladi.

Ikkinchi $P(x) - P(-x) = 4x$ holda $P(x) = \sum_{k=0}^{m} q_k x^k$ deb olsak, unda

$$P(x) - P(-x) = \sum_{k=0}^{m} q_k x^k - \sum_{k=0}^{m} q_k (-x)^k = 2[q_1 x + q_3 x^3 + q_5 x^5 + \cdots] = 4x \Rightarrow$$

$$q_1 = 2, q_3 = q_5 = \cdots = 0 \Rightarrow P(x) = 2x + \sum_{k=0}^{n} p_k x^{2k}. \qquad (7)$$

Bu holda masala shartidan quyidagi tenglikka ega bo'lamiz:

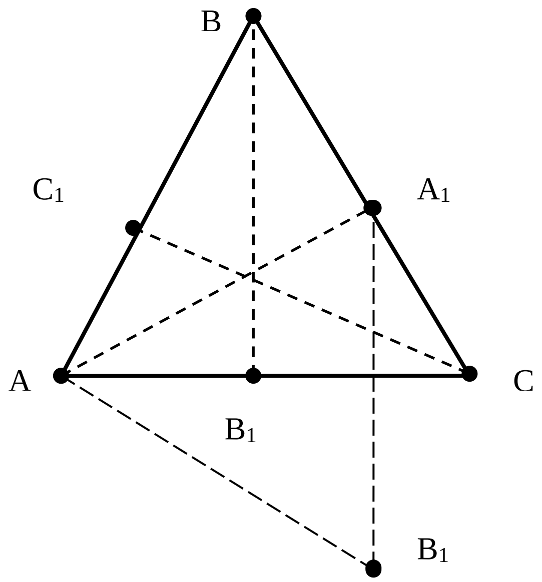
$$P(x^2) + x[4P(x) - 4x] = [P(x)]^2 + 2x^2 \Rightarrow P(x^2) - 2x^2 = [P(x)]^2 - 4xP(x) + 4x^2 \Rightarrow$$

$$\Rightarrow P(x^2) - 2x^2 = [P(x) - 2x]^2 \Rightarrow \sum_{k=0}^{n} p_k x^{4k} = [\sum_{k=0}^{n} p_k x^{2k}]^2 = \sum_{k,j=0}^{n} p_k p_j x^{2(k+j)} \qquad (8)$$

(8) tenglik ustida (3) tenglikdagi singari mulohazalar yuritib, $p_n = 1,\ p_k = 0\ (k = 0,1,...,n-1)$ natijaga kelamiz. Demak, ikkinchi holda masala javobi $P(x) = 2x + x^{2n}, n = 0,1,2,3,\cdots$, ko'rinishda bo'lar ekan.

3-Misol. Agar ABC uchburchakning AA_1 , BB_1 va CC_1 balandliklari uchun $\overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = \vec{0}$ tenglik o'rinli bo'lsa, uchburchakning burchaklarini toping.

Yechim: Masala shartlari muntazam ABC uchburchak ($AB=AC=BC$) uchun bajarilishini ko'rsatamiz. Bu uchburchakning balandliklari $AA_1= BB_1= CC_1$ bo'ladi. Bu balandliklar orasidagi o'tkir burchaklar 60^0 bo'ladi. Bu balandliklardan tuzilgan $\overrightarrow{AA_1}, \overrightarrow{BB_1}, \overrightarrow{CC_1}$ vektorlar yig'indisini uchburchak usulida topamiz.



Buning uchun dastlab $\overrightarrow{BB_1}$ vektorni parallel ko'chirib, uning B boshini $\overrightarrow{AA_1}$ vektorning A_1 uchiga keltiramiz va $\overrightarrow{BB_1} = \overrightarrow{A_1B_1}$ vektorni hosil etamiz. So'ngra $\overrightarrow{CC_1}$ vektorni parallel ko'chirib, uning C boshini $\overrightarrow{A_1B_1}$ vektorning B_1 uchiga keltiramiz. Bunda $\angle AA_1B_1 = 60^0$, BB_1 va CC_1 orasidagi burchak ham 60^0 bo'lgani uchun, $\overrightarrow{CC_1}$ vektorning C_1 uchi A nuqta bilan ustma-ust tushadi va $\overrightarrow{CC_1} = \overrightarrow{B_1A}$ ekanligi kelib chiqadi. Bu holda vektorlarni qo'shishning uchburchak qoidasiga asosan $\overrightarrow{AA_1} + \overrightarrow{A_1B_1} = \overrightarrow{AB_1} = -\overrightarrow{B_1A} \Rightarrow \overrightarrow{AA_1} + \overrightarrow{A_1B_1} + \overrightarrow{B_1A} = \vec{0} \Rightarrow \overrightarrow{AA_1} + \overrightarrow{BB_1} + \overrightarrow{CC_1} = \vec{0}$

.

Bu yerdan va test javobi yagona bo'lishi kerakligidan masala shartini qanoatlantiruvchi ucburchakning burchaklari o'zaro teng va 60^0 ekanligi kelib chiqadi.

Javob: Berilgan ABC uchburchakda $\angle A = \angle B = \angle C = 60^0$.

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