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METHODS OF SOLVING NON-STANDARD PROBLEMS INVOLVING MATRIX AND DETERMINANTS FOR THE STUDENT SCIENCE OLYMPIAD

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ABSTRACT

In this article, convenient ways to solve some non-standard problems found in Science Olympiad problems for students of non-mathematical bachelor's degree in Higher Mathematics are presented.

Keywords: Olympiad, determinant, matrix, vector.

TALABALAR FAN OLIMPIADASIGA DOIR MATRITSA VA DETERMINANT QATNASHGAN NOSTANDART MASALALARNI YECHISH USULLARI

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АННОТАЦИЯ

В данной статье представлены удобные способы решения некоторых нестандартных задач, встречающихся в задачах олимпиады по науке для студентов нематематического бакалавриата по высшей математике.

Ключевые слова: Олимпиада, детерминант, матрица, вектор.

ANNOTATSIYA

Ushbu maqolada Oliy matematika fanidan nomatematik bakalavr yo'nalishi talabalari uchun fan olimpiada masalalarida uchraydigan ba'zi bir nostandart masalalarni yechishning qulay usullari keltirilgan.

Kalit so'zlar: Olimpiada, determinant, matritsa, vektor.

Quyidagilar olimpiadaning asosiy vazifalari hisoblanadi: Iste'dodli talabalarni aniqlash, ular faoliyatini ilmiy va uslubiy jihatdan muvofiqlashtirib borish, sohada ilg'or tajribalarni ommalashtirishda respublika va xalqaro tajribalarni inobatga olish;
iqtidorli talabalarni qo'llab-quvvatlash, intellektual o'sishi uchun sharoit yaratish;
talabalarda o'quv faoliyatiga va bo'lajak kasbiga qiziqishni oshirish; talabalar uchun o'quv mashg'ulotlarini tashkillashtirishda zamonaviy pedagogik va axborot-kommunikatsiya texnologiyalari hamda interfaol usullarni tatbiq etilishini ta'minlash;
Olimpiada gumanitar, tabiiy ilmiy fanlar hamda umumkasbiy, ixtisoslik fanlari bo'yicha alohida-alohida o'tkaziladi. “Gumanitar va tabiiy-ilmiy fanlar” blogi bo'yicha o'tkaziladigan olimpiadada bakalavriatning kunduzgi bo'lim 2-kurs talabalari, “Umumkasbiy hamda ixtisoslik fanlari” bo'yicha 3-, 4-va undan yuqori kurs talabalari ishtirok etadi. Bular o'quvchilarni fanga bo'lgan qiziqishini shakllantirish, faolligini oshirish va ularni

rag'batlantirish bilan bog'liqdir. Olimpiada ishtirokchilarning ijodiy faolligini o'stirishda, tafakkur jarayonlarini shakllantirishda mantiqiy fikrlash, matematikadan nostandart masalalarni yechish alohida ahamiyat kasb etadi.

Iqtidorli talabalarning fanlardan egallagan bilimlari qanchalik chuqur va mustahkamligini, ularning ijodiy fikrlash doirasining kengligini aniqlovchi mezonlardan biri olimpiadadir. Quyida olimoiadaga doir nostandart masalalarni yechish usullarini keltiramiz.

1-Masala. Agar $I+AB$ matritsaning teskarisi mavjud bo'lsa, $I+BA$ martirsaning ham teskarisi mavjud bo'lishini isbotlang. (I -birlik matritsa)

Yechish: Agar $I+AB$ matritsaning teskarisi mavjud bo'lsa, u holda $\det(I+AB)\neq 0$. Bizdan $\det(I+BA)\neq 0$ ekanligini ko'rsatish talab etiladi.

Quyidagi hollar bo'lishi mumkin:

1-hol. A-teskarilanuvchi ya'ni, $\det A\neq 0$.

Ravshanki,

$$I+AB=AA^{-1}+AB=A(A^{-1}+B) \qquad \text{va}$$

$$I+BA=A^{-1}A+BA=(A^{-1}+B)A$$

Bu tengliklardan quyidagiga ega bo'lamiz.

$$\det(I+AB)=\det A(A^{-1}+B)=\det A\cdot\det(A^{-1}+B)$$

$$\det(I+BA)=\det(A^{-1}+B)A=\det(A^{-1}+B)\cdot\det A$$

Demak, $\det(I+BA)=\det(I+AB)\neq 0$.

2-hol. $\det A=0$. Yuqoridagi teoremaga ko'ra, $\exists\lambda_1\ ,\lambda_2,...,\lambda_m\in C$ va $A_\lambda\in\mathbf{C}[m\times m]$ topilib $\forall\lambda\in\mathbf{C}\setminus\{\lambda_1,\lambda_2,...\lambda_m\}$ uchun $\det A\neq 0$ va $\lim\limits_{\lambda\to 0}\det A_\lambda=\det A$ bo'ladi. A_λ teskarilanuvchiligidan, 1-holga ko'ra, $\det(I+A_\lambda B)=\det(I+BA_\lambda)$ tenglik o'rinli.

Determinant uzluksiz funksiya ekanidan foydalanib, $\lambda\to 0$ da limitga o'tamiz va quyidagilarni hosil qilamiz.

$$\lim_{\lambda\to 0}\det(I+A_\lambda B)=\det\lim_{\lambda\to 0}(I+A_\lambda B)=\det(I+A\ B)$$

$$\lim_{\lambda\to 0}\det(I+BA_\lambda)=\det\lim_{\lambda\to 0}(I+BA_\lambda)=\det(I+BA\ )$$

Demak, $\det(I+BA)=\det(I+AB)\neq 0$

Yuqoridagi natijalardan $\det(I+BA)\neq 0$ ekanligi kelib chiqdi. Bu esa $I+BA$ matritsaning teskarilanuvchiligini bildiradi. Isbot tugadi.

2-Masala. Quyidagi limitni hisoblang

$$\lim_{x\to 0}\left(\lim_{n\to\infty}\frac{1}{x}\left(A^n-E\right)\right)$$

Bu yerda, $A=\begin{pmatrix} 1 & \frac{x}{n} \\ -\frac{x}{n} & 1 \end{pmatrix}$, $E=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Yechish:

$$\begin{vmatrix} 1-\lambda & \frac{x}{n} \\ -\frac{x}{n} & 1-\lambda \end{vmatrix}=0$$

$$(\lambda-1)^2+\frac{x^2}{n^2}=0$$

$$\lambda-1=\pm i\frac{x}{n}$$

$\lambda_1=1+i\frac{x}{n}$, $\lambda_2=1-i\frac{x}{n}$ xos qiymatlarni $\lambda_1=1+i\frac{x}{n}$ xos qiymatga mos keluvchi xos vektorni topamiz:

$$\begin{pmatrix} -i\frac{x}{n} & \frac{x}{n} \\ -\frac{x}{n} & -i\frac{x}{n} \end{pmatrix}\cdot\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-i\frac{x}{n}a_1+\frac{x}{n}a_2=0$$

$$a_2=ia$$

$\bar{e}_1=\begin{pmatrix} 1 \\ i \end{pmatrix}$ xos vektorlar bo‘ladi.

$\lambda_2=1-i\frac{x}{n}$ xos qiymatga $\bar{e}_2=\begin{pmatrix} 1 \\ -i \end{pmatrix}$ xos vektor mos keladi. Ushbu $C=\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$ matritsani tuzib olamiz. $C^{-1}=\frac{1}{2}\cdot\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$ bo‘ladi.

$C^{-1}\cdot A\cdot C=\frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\cdot\begin{pmatrix} 1 & \frac{x}{n} \\ -\frac{x}{n} & 1 \end{pmatrix}\cdot\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}=\frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}\cdot\begin{pmatrix} 1+i\frac{x}{n} & 1-i\frac{x}{n} \\ -\frac{x}{n}+i & -\frac{x}{n}-i \end{pmatrix}=\begin{pmatrix} 1+i\frac{x}{n} & 0 \\ 0 & 1-i\frac{x}{n} \end{pmatrix}=J$ Demak,

$A=C\cdot J\cdot C^{-1}$

$$A^2=C\cdot J\cdot C^{-1}\cdot C\cdot J\cdot C^{-1}=C\cdot J^2\cdot C^{-1},$$

$A^k=C\cdot J^k\cdot C^{-1}$ bo‘lsin deb faraz qilib, $A^{k+1}=C\cdot J^{k+1}\cdot C^{-1}$ bo‘lishini ko‘rsatamiz:

$A^{k+1}=A^k\cdot A=C\cdot J^k\cdot C^{-1}\cdot C\cdot J\cdot C^{-1}=C\cdot J^{k+1}C^{-1}$

Demak, ixtiyoriy n uchun

$$A^n = C\cdot J^n \cdot C^{-1} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\cdot \begin{pmatrix} \left(1+i\frac{x}{n}\right)^2 & 0 \\ 0 & \left(1-i\frac{x}{n}\right)^2 \end{pmatrix}\cdot \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$$

$$B = \lim_{n\to\infty} A^n = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\cdot \begin{pmatrix} e^{ix} & 0 \\ 0 & e^{-ix} \end{pmatrix}\cdot \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \lim_{n\to\infty}\left(\lim_{n\to\infty}\frac{1}{x}\left(A^n - E\right)\right) = \lim_{n\to\infty}\frac{1}{x}\left(B - C\cdot C^{-1}\right) =$$

$$C\cdot \lim_{n\to\infty}\begin{pmatrix} \frac{e^{ix-1}}{x} & 0 \\ 0 & \frac{e^{-ix-1}}{x} \end{pmatrix}\cdot C^{-1} = C\cdot \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\cdot i^{-1} = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\cdot \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\cdot \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} =$$

$$\frac{1}{2}\begin{pmatrix} i & -i \\ -1 & -1 \end{pmatrix}\cdot \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \frac{1}{2}\cdot \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Javob: $\left\{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right\}$

3-Masala. $\forall A, B \in \mathbf{C}[m\times m]$ matritsalar uchun quyidagi tenglik o‘rinli ekanini isbotlang.

$$\det(I + AB) = \det(I + BA)$$

Yechish: Quyidagi hollarni qaraymiz:

1-hol. A-teskarilanuvchi ya’ni, $\det A \neq 0$.

Ravshanki, $I + AB = AA^{-1} + AB = A(A^{-1} + B)$ va

$$I + BA = A^{-1}A + BA = (A^{-1} + B)A$$

Bu tengliklardan quyidagiga ega bo‘lamiz.

$$\det(I + AB) = \det A(A^{-1} + B) = \det A \cdot \det(A^{-1} + B)$$

$$\det(I + BA) = \det(A^{-1} + B)A = \det(A^{-1} + B)\cdot \det A$$

Demak, $\det(I + AB) = \det(I + BA)$.

2-hol. $\det A = 0$. Yuqoridagi teoremaga ko‘ra, $\exists \lambda_1, \lambda_2, \ldots, \lambda_m \in C$ va $A_\lambda \in \mathbf{C}[m\times m]$ topilib $\forall \lambda \in \mathbf{C} \setminus \{\lambda_1, \lambda_2, \ldots \lambda_m\}$ uchun $\det A \neq 0$ va $\lim_{\lambda\to 0}\det A_\lambda = \det A$ bo‘ladi. A_λ teskarilanuvchiligidan, 1-holga ko‘ra, $\det(I + A_\lambda B) = \det(I + BA_\lambda)$ tenglik o‘rinli.

Determinant uzluksiz funksiya ekanidan foydalanib, $\lambda \to 0$ da limitga o‘tamiz va quyidagilarni hosil qilamiz.

$$\lim_{\lambda\to 0}\det(I + A_\lambda B) = \det\lim_{\lambda\to 0}(I + A_\lambda B) = \det(I + A\,B)$$

$$\lim_{\lambda\to 0}\det(I + BA_\lambda) = \det\lim_{\lambda\to 0}(I + BA_\lambda) = \det(I + BA\,)$$

Demak, $\det(I + AB) = \det(I + BA)$

4-Masala. Agar $A=\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}$ bo'lsa, e^A ni hisoblang.

Yechish: $\begin{pmatrix}x'\\ y'\end{pmatrix}=\begin{pmatrix}3 & -1\\ 2 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$ differensial tenglamani qaraymiz. Uning xarakteristik tenglamasini yechamiz. $\det(A-\lambda I)=\begin{vmatrix}3-\lambda & -1\\ 2 & -\lambda\end{vmatrix}=\lambda^2-3\lambda+2=0\Rightarrow\lambda_1=1,\ \lambda_2=2$. Endi har bir xos qiymatga mos bo'lgan xos vektorni aniqlaymiz.

$$(A-1\cdot I)v_1=\begin{pmatrix}2 & -1\\ 2 & -1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=0\Rightarrow 2a-b=0\Rightarrow a=1,\ b=2\Rightarrow v_1=\begin{pmatrix}1\\ 2\end{pmatrix}$$

$$(A-2\cdot I)v_2=\begin{pmatrix}1 & -1\\ 2 & -2\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=0\Rightarrow a-b=0\Rightarrow a=1,\ b=1\Rightarrow v_2=\begin{pmatrix}1\\ 1\end{pmatrix}$$

Sistemaning umumiy yechimi quyidagi ko'rinishda bo'ladi:

$u(t)=\begin{pmatrix}x\\ y\end{pmatrix}=C_1\begin{pmatrix}1\\ 2\end{pmatrix}e^t+C_2\begin{pmatrix}1\\ 1\end{pmatrix}e^{2t}=\begin{pmatrix}C_1e^t+C_2e^{2t}\\ 2C_1e^t+C_2e^{2t}\end{pmatrix}$ bunga $t=0$ ni qo'yib, $u(0)=\begin{pmatrix}x(0)\\ y(0)\end{pmatrix}=\begin{pmatrix}C_1+C_2\\ 2C_1+C_2\end{pmatrix}$ ni hosil qilamiz. Natijada quyidagilarga ega bo'lamiz:

$$u(0)=e_1\Leftrightarrow\begin{pmatrix}C_1+C_2\\ 2C_1+C_2\end{pmatrix}=\begin{pmatrix}1\\ 0\end{pmatrix}\Rightarrow\begin{cases}C_1=-1\\ C_2=2\end{cases}\Rightarrow u_1(t)=\begin{pmatrix}2e^{2t}-e^t\\ 2e^{2t}-2e^t\end{pmatrix}$$

$$u(0)=e_2\Leftrightarrow\begin{pmatrix}C_1+C_2\\ 2C_1+C_2\end{pmatrix}=\begin{pmatrix}0\\ 1\end{pmatrix}\Rightarrow\begin{cases}C_1=1\\ C_2=-1\end{cases}\Rightarrow u_2(t)=\begin{pmatrix}e^t-e^{2t}\\ 2e^t-e^{2t}\end{pmatrix}$$

$u_1(t)$ va $u_2(t)$ vektorlardan matritsa tuzamiz. $e^{tA}=\begin{pmatrix}2e^{2t}-e^t & e^t-2e^{2t}\\ 2e^{2t}-2e^t & 2e^t-e^{2t}\end{pmatrix}$ bunga $t=1$ ni qo'ysak,

$e^A=\begin{pmatrix}2e^2-e & e-2e^2\\ 2e^2-2e & 2e-e^2\end{pmatrix}$ biz izlayotgan matritsa hosil bo'ladi.

5-Masala. a_{ij} koeffitsientlar butun son bo'lsa, quyidagi tenglamalar sistemasi yagona yechimga ega bo'lishini ko'rsating.

$$\begin{cases}\frac{1}{2}x_1=a_{11}x_1+\quad+a_{1n}x_n\\ \frac{1}{2}x_2=a_{21}x_1+\quad+a_{2n}x_n\\ \dots\dots\dots\dots\dots\dots\dots\\ \frac{1}{2}x_n=a_{n1}x_1+\quad+a_{nn}x_n\end{cases}$$

Yechish: Sistemaning determinanti

$$D=\begin{vmatrix} a_{11}-\frac{1}{2} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\frac{1}{2} & & a_{2n} \\ \dots\dots\dots & \dots\dots\dots\dots & & \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & & & \\ a_{n1} & a_{n2} & & a_{nn}-\frac{1}{2} \end{vmatrix}$$

Agar biz

$$P(\lambda)=\begin{vmatrix} a_{11}-\lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22}-\lambda & & a_{2n} \\ \dots\dots\dots & \dots\dots\dots\dots & & \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & & & \\ a_{n1} & a_{n2} & & a_{nn}-\lambda \end{vmatrix}$$

belgilash kiritsak, u holda $D=P\left(\frac{1}{2}\right)$ bo‘ladi. Ikkinchi tomondan

$$P(\lambda)=(-1)^n\lambda^n+b_1\lambda^{n-1}+\dots+b_n$$

ko‘rinishda bo‘ladi, bu yerda b_i – butun son $i=\overline{1,n}$

Agarda $P\left(\frac{1}{2}\right)=0$ bo‘lsa, u holda

$$(-1)^n\frac{1}{2^n}+b_1\frac{1}{2^{n-1}}+\dots+b_n=0$$

va bu oxirgi tenglikni ikkala tarafini ham 2^n ga ko‘paytirsak,

$$(-1)^n+2b_1+2^2b_2\dots+2^nb_n=0$$

bo‘ladi.

Ushbu $2b_1+2^2b_2\dots+2^nb_n=N$ belgilash kiritsak, u holda

$$(-1)^n+2N=0$$

ega bo‘lamiz. Bunday bo‘lishi mumkin emas, chunki N butun.

Demak, $D=P\left(\frac{1}{2}\right)\neq 0$ ekan, u holda sistema yagona yechimga ega va bu yechim

$$x_1=x_2=\dots=x_n=0 \text{ bo‘ladi.}$$

REFERENCES

1. Quromboyev, H. (2022). Nostandart olimpiada masalalarini yechish usullari haqida. Академические исследования в современной науке, 1(13), 231-233.
2. Қуромбоев, Ҳ. (2022). I тип зигел соҳаси учун карлеман формуласи. Models and methods in modern science, 1(13), 52-56.
3. Куромбоев, Х. Н. (2019). МАТЕМАТИЧЕСКАЯ НАУКА К ПОВСЕДНЕВНОЙ ЖИЗНИ. Экономика и социум, (2), 619-621.
4. Куромбоев, Х. Н. (2020). О ФОРМУЛЕ КАРЛЕМАНА. Мировая наука, (5), 278-282.
5. Kuromboev, K. N., Allanazarov, K., & Shokirov, J. (2019). INDIVIDUAL ABILITIES OF STUDENTS IN TEACHING MATHEMATICS. Экономика и социум, (2), 622-625.
6. Куромбоев, Х. Н., Муродхужаев, Р., & Носирхужаев, Н. (2019). ПОНЯТИЕ КОМПЛЕКСНЫХ ЧИСЕЛ. Экономика и социум, (2), 625-626.
7. Kuromboev, K. N. (2019). HISTORY AND APPLICATION OF THE MATRIX. Экономика и социум, (2), 615-618.
8. Usmonov, B. Z., & Qobilov, T. A. (2021). Isbotlashlarda taqqoslamalar ning o'rni. Academic research in educational sciences, 2(5), 2181-1385.
9. Kutlimurotov, R. A., Usmonov, B. Z., Toshbayeva, N. Y. L., & Eshqorayev, Q. (2021). CHEKLI ZANJIRLI KASRLARNI BAZI MASALALARGA TADBIQI. Academic research in educational sciences, 2(5), 914-921.
10. Eshqoraev, Q. (2021). VERBAL INTELLEKT–KASBIY VA IJTIMOIY KOMTETENTLIKNI RIVOJLANTIRISH OMILI SIFATIDA. Academic research in educational sciences, 2(CSPI conference 3), 428-432.
11. Тошбоева, Н. (2021). Развитие творческих способов студентов с помощью нестандартных вопросов. Academic research in educational sciences, 2(NUU Conference 1), 393-395.
12. Кобилов, Т. А. Экстент гиперпространства сходящейся под последовательностей. MINISTRY OF HIGHER AND SECONDARY SPECIAL EDUCATION OF THE REPUBLIC OF UZBEKISTAN, 135.
13. Ergashev, I. (2022). QIZIQARLI GEOMETRIK MASALALARNI YECHISHDA KREATIV YONDASHUV. Models and methods in modern science, 1(13), 90-92.
14. Исломов, С. М. d-сепарабельность гиперпространств. In РЕСПУБЛИКАНСКОЙ НАУЧНОЙ КОНФЕРЕНЦИИ (p. 154).
15. Mashal-ogli, I. S. A. (2022, August). CLUSTER APPROACH: USING IN MATHEMATICS TEACHING: Islomov San'at Mashal-ogli Chirchik State Pedagogical Institute of Tashkent Region, Uzbekistan, email: islomovsanat9313@ gmail. com. In Научно-практическая конференция.
16. qizi Rustamova, S. A. (2022, September). INTERFAOL METODLAR ORQALI TALABALAR FAOLLIGINI OSHIRISH. In INTERNATIONAL CONFERENCES (Vol. 1, No. 11, pp. 41-46).
17. Abdullayev, S. A., Aktamov, F., & Raupova, M. (2021). “FUNKSIYA XOSILASI” MAVZUSINI O ‘RGANISHDA KLASTER MODELIDAN FOYDALANISH METODIKASI. Academic research in educational sciences, 2(CSPI conference 3), 420-424.

18. Abdullayev, S. A. O. G. L., & Ahmadjonova, M. A. Q. (2021). MATLAB TIZIMIDA ODDIY DIFFERENSIAL TENGLAMALARNI YECHISH. Academic research in educational sciences, 2(11), 1576-1584.
19. Abdullayev, S. A., Aktamov, F., & Raupova, M. (2021). “Funksiya xosilasi” mavzusini o ‘rganishda klaster modelidan foydalanish metodikasi. Academic research in educational sciences, 2(CSPI conference 3), 420-424.
20. Murtozaqulov Z. M. O. G. L., Solayeva M. N. darslikdagi differensial tenglamalarni yechishdagi yetishmayotgan metodlar va ma’lumotlar //Academic research in educational sciences. – 2021. – Т. 2. – №. CSPI conference 3. – С. 462-467.
21. MURTOZAQULOV Z. M., ABDUJABBOROV S. H. F. Tenglamalar sistemasini yechishda qulay bo’lgan metod va ko’rsatmalar //ЭКОНОМИКА. – С. 898-904.
22. Сейтов, А. Ж., Ханимқулов, Б. Р., Гаипов, М. А., & Юсупов, М. Р. (2021). Зарфшон дарёси оқимининг ҳосил бўлишига атмосфера ёғинлари ва ҳаво ҳароратининг таъсири. Academic research in educational sciences, 2(5), 156-162.
23. Solayeva, M. N., Yusupov, M. R., Abdullayev, Sh. A. (2022). Ba'zi bir ajoyib limitlarga oid misollarni noananaviy uslublardan foydalanib yechish usullari. TABIIY-ILMIY FANLARNI O‘QITISHDA FUNDAMENTAL VA AMALIY YONDASHUVLAR Respublika ilmiy anjuman materiallari to‘plami, 1(1), 164-168.
24. Mahkamov, E. M., & Quljonov, N. J. (2021). FINLANDIYA O ‘RTA TA’LIM MAKTABLARIDA O ‘QITISHNI TAHLIL QILISH (MATEMATIKA FAN MISOLIDA). Academic research in educational sciences, 2(CSPI conference 1), 146-149.