HAWKING THERMAL RADIATION AND LIFETIME OF BLACK HOLES IN BRANEWORLD

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ANNOTATION

The thermodynamic properties of the primordial black hole, (PBH), in the braneworld are investigated. Hawking radiation and the life time of PBHs have been studied. An exact analytical expression for the Bekenstein-Hawking's entropy, the temperature and heat capacity have been found. Time dependence of thermodynamic quantities of PBH has been studied. It is argued that the life time of PBHs in the early Universe is shortened at least for one order which lead to their evaporation. It can be a reason why we do not observe final rapid evaporation of PBHs in the recent epoch of the Universe.

Keywords: Primordial black hole; Braneworld; Hawking radiation; Life time.

INTRODUCTION

It is widely believed that PBHs are a hypothetical type of black hole that have been formed in the initial period of the universe (over the second after the Big Bang), in the named as radiation dominated era. There are several formation mechanisms of PBHs suchlike, because of inflation[1], collapse of large density fluctuation (high densities and inhomogeneous conditions of the matter can make such dense regions because of gravitational collapse)[2], phase transition[3], cosmic string loops [4] or due to bubble collisions.

The existence of similar black holes has been first suggested by authors [5], later the theory of PBHs is extended by S. Hawking PBHs are not created from stellar gravitational collapse, their masses can be much more smaller or much more larger than stellar mass. Their mass is approximately equal to the horizon mass, that can be expressed as [6]

$$M \sim \frac{c^3 t}{G} \sim 10^{15}\left(\frac{t}{10^{-23} s}\right) g$$

In the opinion by Hawking that the Plank mass, which is appropriate to the Plank time scale $t_{Pl} \sim 10^{-43} s$, would be approximately $M \geq 10^{-5} g$, while the supermassive black hole (SMBH) mass $M \sim 10^6 M_{\odot}$ can occur within a second $\sim 1s$ after the Big Bang.

According to Hawking, PBHs radiate thermally and evaporation timescale is calculated by [7]

$$\tau \sim \frac{\hbar c^4 t}{4\pi G^2} M^3 \sim 10^{10}\left(\frac{M}{10^{15} g}\right) Gyr$$

However, the life time of PBHs with a mass around 10^{13}g is approximately equal to the age of the present universe.

One may say that PBHs included in to the class of massive compact halo objects (MACHOs). The abundance of PBHs might explication dark matter (DM) problem, which is one most significant and interesting subjects of present cosmology and astrophysics. Nevertheless, tight limits on their abundance have been set up from different astrophysical and cosmological observations and it is now excluded that they contribute sufficiently to DM over most of the credible mass range. Even so, they are as well good nominees for being the seeds of the supermassive black holes, with mass range $M \sim 10^6 - 10^{10} M_{\odot}$, at the center of massive galaxies, as well as of intermediate-mass black holes $M \sim 50 - 100^6 M_{\odot}$.

In accordance with result by Advanced LIGO/VIRGO on observation of gravitational waves emitted by the merger of binary system of the black holes with $\sim 30 M_{\odot}$ masses, three different groups of scientists severally concluded that the detected black holes had a primordial origin. [8] Two of these groups have observed that the merging rates of the binary system inferred by LIGO are consistent with a scenario in which all DM in the Universe is made of PBHs. But, the third group has alleged that the merging rates of the system are incompatible with all DM scenario and that PBHs could only contribute to less than $\sim 1\%$ of the total DM. The unexpected large mass of the black holes detected by LIGO has strongly revived interest in PBHs with masses in the range $1 \leq M / M_{\odot} \leq 100$ It is however still debated whether this range is excluded or not by other observations, such as the absence of microlensing of stars the cosmic microwave background anisotropies the size of faint dwarf galaxies, and the absence of correlation between X-ray and radio sources towards the galactic center.

The aim of the present work is to study the Hawking radiation and the life time of the PBHs in the braneworld scenario. The braneworld model is first proposed by Rundall and Sundrum, the main idea of this model is that matter is restricted to three dimensional brane, inserted in a larger space, so-called bulk, in which only gravitation interaction can propagate. the spherically symmetric and static exterior vacuum solutions of the braneworld models has been obtained.

The paper is organized as follows. Sect. 2 is devoted to the study of the life time of PBHs in the braneworld. In Sect. 3, we present the basic equations for the description of the PBH thermodynamics in the braneworld. In Sec 4 we present emission rate and gray-body factor of PBHs in the braneworld. The summary of the obtained results is reported in Sect. 5. Throughout the paper we employ the convention of a metric with signature $(-,+,+,+)$. We use units in which $G = c = k = \hbar = 1$, but we restore them when we have to compare our findings with observational data. Greek indices run from 0 to 3, while Latin indices run from 1 to 3.

LIFE TIME OF PBH IN THE BRANEWORLD

In this section we study the life time of the PBHs in the braneworld and evaluate the effect of the brane tension on the life time of the static black hole. In the spherical coordinates (t, r, θ, ϕ) the spacetime metric for a spherical symmetric, static black hole in the braneworld is given by [9]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\phi^2) \tag{1}$$

with

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^*}{r^2} \tag{2}$$

where *M* is the total mass of PBH and Q^* is the brane charge parameter, sometimes it is called "Weyl charge", in fact that the brane charge parameter has a negative value ($Q^* < 0$). To be convenient for later calculations, here we introduce the new positive brane parameter, i.e. $b^2 = -Q^*$, the sign of last term in metric function *f(r)* will be changed.

The radius of an outer horizon of black hole can be calculated by condition, i.e. *f = 0*, that is

$$r_+ = M + \sqrt{M^2 + b^2} \geq 2M \tag{3}$$

which is greater then Schwarzschild radius. Recalling equation (3) Hawking temperature over the surface of the black hole in the braneworld can be obtained as

$$T = \frac{\sqrt{M^2 + b^2}}{2\pi\left(M + \sqrt{M^2 + b^2}\right)^2} \leq \frac{1}{8\pi M} \tag{4}$$

Most important parameter of PBH is it's lifetime. The rate of energy loss from PBH in the braneworld can be approximated with the Stephan-Boltzmann radiation law as

$$\frac{dM}{dt} \sim \sigma A T^4 = 4\pi\sigma r_+^2 T^4 \tag{5}$$

where $\sigma = \pi^2/60$ is the Stephan-Boltzmann constant, A is the surface area of black hole horizon. Using the expressions for the temperature (7) and radius of horizon (6) one can rewrite equation (8) in the form

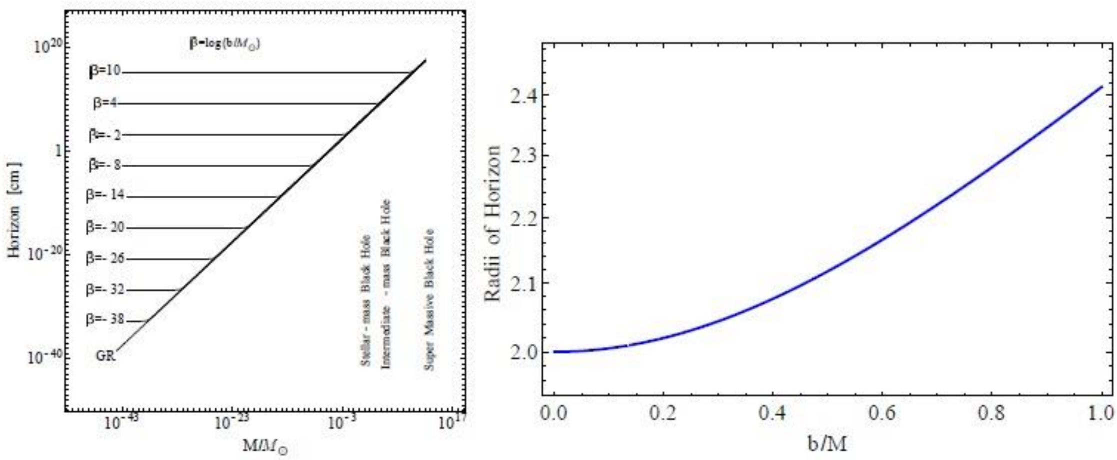

Fig. 1. Left panel: Dependence of radius of horizon of PBH form it's mass for the different values of brane tension. Right panel: Dependence of radius of horizon of PBH form brane tension parameter b/M.

Активация

$$\frac{dM}{dt} = -\frac{1}{240\pi}\frac{\left(M^2+b^2\right)^2}{\left(M+\sqrt{M^2+b^2}\right)^6} \tag{6}$$

now one can easily calculate the life time of PBH in terms of brane charge parameter:

$$\tau = -240\pi\int_M^0 \frac{dM}{\left(M^2+b^2\right)^2}\left(M+\sqrt{M^2+b^2}\right)^6 \tag{7}$$

Before evaluating the integral (10), note that here two possibilities can be observed, first one is parameter b can be assumed as a constant b=const, independently from mass of PBH and in the second case the ratio of brane charge parameter and black hole mass can be constant b/M=const. If we evaluate integral (10) from M to 0 it will give a life time τ of PBH and in case when b=const one can get

$$\tau = 2560\pi M^3\left[1-\frac{3b^3}{2M^2}-\frac{3b^4}{64M^2(b^2+M^2)}+\frac{16M^4-41b^4+16b^2M^2}{16M^3\sqrt{b^2+M^2}}+\frac{105b^3}{64M^3}\tan^{-1}\left(\frac{M}{b}\right)\right] \tag{8}$$

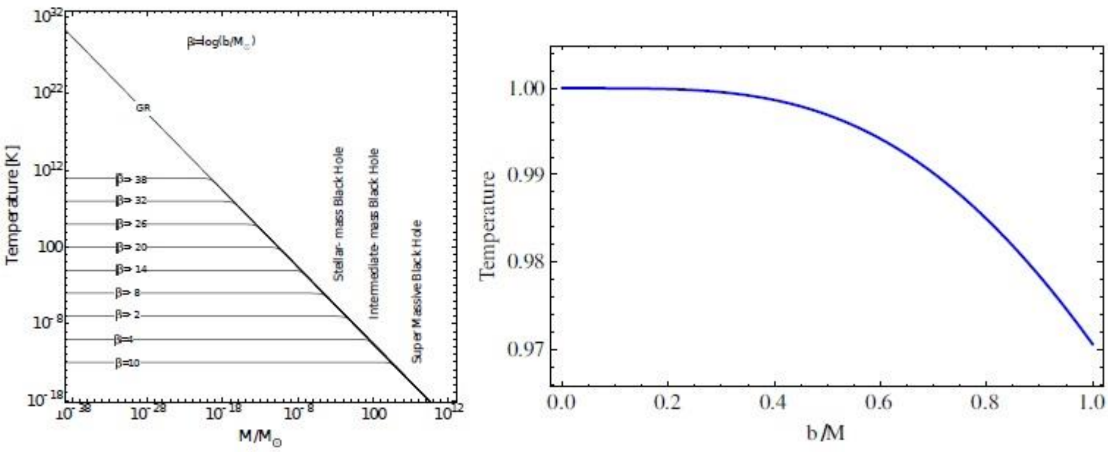

Fig. 2. Left panel: Dependence of temperature of PBH from it's mass for the different values of brane tension. Right panel: Dependence of temperature of PBH form brane tension parameter b/M.

while in case when b/M = const one can obtain

$$\tau = 80\pi M^3\frac{\left(1+\sqrt{1+b^2/M^2}\right)^6}{\left(1+b^2/M^2\right)^2} \tag{9}$$

Note that of both cases the general relativistic limit of life time takes the standard value as obtain in ref [10]

$$\tau_0 = \lim_{b\to 0}\tau = 5120\pi M^3 \tag{10}$$

For small brane charge parameter ratio of lifetime of PBH takes a form

$$\tau = \tau_0\left(1-\frac{3b^2}{2M^2}+\frac{105\pi b^3}{256M^3}\right)+O\left(b^4\right) \tag{11}$$

which shows that the lifetime of PBH will be shorter in the braneworld scenario.

Figure 3 draws the fractional lifetime for various values of brane charge parameter to that of a Schwarzschild black hole (i.e. b=0). The dependence on b clearly influences the lifetime of such black holes. One can easily see that in case of when b=const a value of b=0.5M decreases the value of life time of PBH by roughly 30% percent, while b=M= const case it reaches up to 10% percent.

CONCLUSIONS

In this paper thermodynamic properties of the PBH, in the braneworld in particular the Hawking radiation and evaporation of the PBHs in the braneworld have been studied. Our main results can be summarized as follows:

We have studied the thermodynamic properties of PBH in the braneworld model. In the presence of brane tension an exact analytical expression for the thermodynamic quantities such as the temperature, entropy, Bekenstein-Hawking's entropy, specific heat, heat capacity and free energy have been found.

We have studied the Hawking radiation and the life time of PBHs. It is argued that the life time of PBHs in the early universe is shortened at least for one order which has lead to their early evaporation. It can be a reason why we do not observe evaporation of PBHs as specific high energy bursts in the recent epoch of the universe.

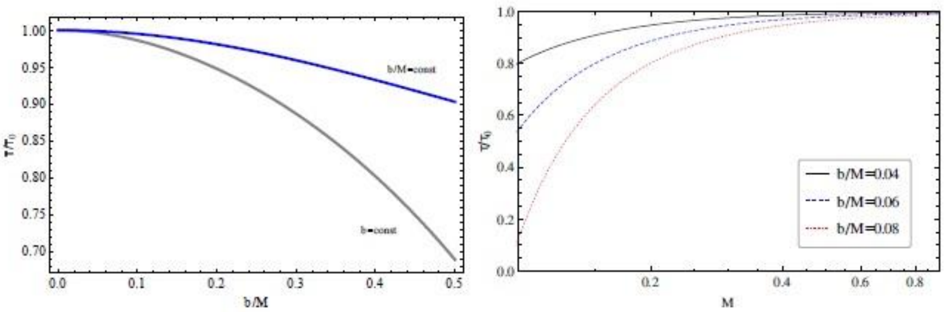

Fig. 3. (Top panel) Fractional lifetime of PBH is a function of brane charge parameter b/M.(Bottom panel) Fractional lifetime of PBH is a function of its mass for different values of brane charge parameter.

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