SOME WAYS TO FIND INTEGRALIZING MULTIPLICATORS

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ABSTRACT

The article presents some special methods of the science of differential equations for solving complete differential equations and finding integral multipliers. The convenience and effectiveness of these methods are revealed.

Keywords: differential equations, complete differential, integrating multiplier, function, sphere.

INTEGRALLOVCHI KO'PAYTUVCHI TOPISHNING BA'ZI USULLARI

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ANNOTATSIYA

Maqolada Differensial tenglamalar fanining to'liq differensial tenglamalar yechish va integral ko'paytuvchilarni topishni topishning ba'zi bir xususiy usullari hadida ma'lumotlar keltirilgan. Bu usullarni qulayliklari va samarali tomonlari ochib berilgan.
Kalit so'zlar: differensial tenglamalar, to'liq differnsial, integrallovchi ko'paytuvchi, funksiya, soha.
Аннотация: В статье представлены некоторые специальные методы науки о дифференциальных уравнениях для решения полных дифференциальных уравнений и нахождения интегральных множителей. Выявлены удобство и эффективность этих методов.

Ключевые слова: дифференциальные уравнения, полный дифференциал, интегрирующий множитель, функция, сфера.

Pedagogik institute tabalarini differensial tenglamalar fanini to'liq differential tenglamalar va integral ko'paytuvchi mavzusini o'qitishda ba'zi bir qiyinchiliklar uchrab turadi. Ya'ni talabalar differensial tenglama to'liq differensial tenglama bo'lmasa uni to'liq differensial tenglamaga keltirishda integral ko'paytuvchini topishda muommalarga duch keladi. Bu muommolarni hal qilishda integral ko'paytuvchini topishning umumiy usuli mavjud bo'lmagani uchun uning topishning xususiy hollari juda kata yordam beradi. Ushbu ishda integral ko'paytuvchini topishni talabalar uchun qulayliklar keltiradiga xususiy hollarini keltirib o'tamiz.

Bundan tashqari bu maqolada fanlararo integratsiyalarga ham kata e'tibor qaratilgan, ya'ni differensial tenglamalar va matemayik analiz fanlari orasidagi integratsiyalardan foydalanilgan. [1],[4],[5],[6],[7],[8],[9],[10],[11],[12] ishlarda ham fanlararo ingratsiyalardan keng foydalanilgan.

Birinchi navbatda to'liq differensial tenglama haqida umumiy tuhunchalar, ta'rif va teorimalarni keltirib o'tamiz.

G sohada aniqlangan birorta ham $U(x, y)$ funksiya uchun $\boldsymbol{dU(x,y) = M(x,y)dx + N(x,y)dy}$ tenglik o'rinli bo'lmasin ya'ni $M(x,y)dx + N(x,y)dy = 0$ differensial tenglama to'liq differensialli bo'lmasin.

1-ta'rif. Agar G sohada berilgan $\boldsymbol{M(x,y)}$, $\boldsymbol{N(x,y)}$ va biror $\boldsymbol{\mu(x,y) \neq 0}$ funksiyalar uchun ushbu

$$\boldsymbol{\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0}$$

Tenglama to'liq differensialli bo'lsa $\boldsymbol{M(x,y)dx + N(x,y)dy = 0}$ differensial tenglama to'liq differensialliga keltiriladigan tenglama $\mu(x,y)$ funksiya esa uning integrallovchi ko'paytuvchisi deyiladi.

Bundan keyin yuritiladigan mulohazalar ko'rsatadiki, $M(x,y)$ *va* $N(x,y)$ funksiyalar G sohada differensiallanuvchi bo'lsa , integrallovchi ko'paytuvchi $(x_0, y_0) \in$G nuqtaning yetarlicha kichik atrofida albatta mavjud bo'ladi.

1-teorema. Agar $0 \neq \mu(x,y) \in C^1(G), M(x,y) \in C^1(G), N(x,y) \in C^1(G)$ bo'lib $y = y(x)$, $y(x_0) = y_0)$ funksiya I intervalda aniqlangan hamda $\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$ tenglamaning yechimi bo'lsa, u holda usha funksiya $M(x,y)dx + N(x,y)dy = 0$ tenglamaning ham shu I intervalda aniqlangan yechimi bo'ladi .

Isbot. Shartga ko'ra , $\mu(x,y(x)) \neq 0, x \in I$ *va* $y(x)$ *funksiya* $\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$ ushbu tenglamaning yechimi bo'ladi. Demak, ushbu
$\mu(x,y(x))M\big(x,y(x)\big)dx + \mu\big(x,y(x)\big)N\big(x,y(x)\big)y'(x) \equiv 0, x \in I$ ayniyat o'rinli.Undan
$M\big(x,y(x)\big) + N\big(x,y(x)\big)y'(x) \equiv 0 \;\; x \in I$ ayniyat kelib chiqadi. Bu esa $y(x)$ funksiya $M(x,y)dx + N(x,y)dy = 0$ tenglamaning yechimi ekanligini bildiradi.

Endi integrallovchi ko'paytuvchini to'laroq o'rganamiz

$$\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0$$

tenglama to'liq differensialli bo'lsin. U holda G sohada $\frac{\partial(\mu(x,y)M(x,y))}{\partial y} \equiv \frac{\partial(\mu(x,y)N(x,y))}{\partial x}$ $x, y \in G$ ayniyat o'rinli. Bunday hosilalarni hisoblasak

$$M\frac{\partial\mu}{\partial y} + \mu\frac{\partial M}{\partial y} = N\frac{\partial\mu}{\partial x} + \mu\frac{\partial N}{\partial x} \quad \text{yoki}$$

$$M\frac{\partial\mu}{\partial y} - N\frac{\partial\mu}{\partial x} = \mu(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}) \;\; \text{yoki} \;\; \mu(x,y) > 0, \;\; (x,y) \in G \;\; desak,$$

$$M\frac{\partial ln\mu}{\partial y} - N\frac{\partial ln\mu}{\partial x} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

munosabatga kelamiz. Bu $ln\mu(x,y)$ *funksiyaga* nisbatan birinchi tartibli xususiy hosilali bir jinsli bo'lmagan differensial tenglama. Biz uchun shu tenglamaning biror xususiy yechimini bilish etarli .Bunday yechim $(x_0, y_0) \in G$ nuqtaning yetarlicha kichik atrofida $M, N, \frac{\partial N}{\partial x}, \frac{\partial M}{\partial y}$ G sohada uzluksiz bo'lgani uchun mavjud.

2-teorima Agar $M(x,y)dx + N(x,y)dy = 0$ differensial tenglama $U(x,y) = c$ umumiy integralga ega bo'lsa, u holda bu tenglama uchun integrallovchi ko'paytuvchi mavjud bo'ladi.

Integrallovchi ko'paytuvchini topishning bazi xususiy hollariga to'xtalamiz.

$\mu(x,y) \neq 0, \mu(x,y) \neq const$. Integrallovchi ko'paytuvchi faqat x *yoki* y ning funksiyasi bo'lgan hollar eng sodda holler hisoblanadi.

a) $\mu(x,y) = \mu(x)$ bo'lsin.Bunda yuqoridagi tenglama soddalashadi (chunki $\frac{\partial ln\mu}{\partial y} = 0$):

$$-N\frac{\partial ln\mu}{\partial x} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

$\frac{\partial ln\mu}{\partial x} = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{N}$. $\mu(x,y)$ funksiya uchun yuqorida qilingan farazning o'ng tomoni faqat x ning funksiyasi bo'lishidan iboratdir.Yuqorida tenglamaning ikki tomonini x_0 dan x gacha integrallaymiz: $\mu(x) = Ce^{\int_{x_0}^{x} \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx}$ Bizni birorta integrallovchi ko'paytuvchi qiziqtirayotgani uchun

$C = 1$ desa bo'ladi.

b) Endi $\mu(x,y) = \mu(y)$ bo'lsin, yuqoridagi tenglama bunday ko'rinishga keladi:

$$N\frac{\partial ln\mu}{\partial y} = \frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}.$$

Undan y_0 *dan* y gacha integrallash natijasida $(x, y_0) \in G$, $(x,y) \in G$ $\mu(y) = Ce^{\int_{y_0}^{y} \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{N} dy}$ ifodani topamiz.

Misollar. 1. Ushbu $\frac{dy}{dx} = a(x)y + b(x)$

Chiziqli differensial tenglama berilgan bo'lsin. *Uni* $[a(x)y + b(x)]dx - dy = 0$ ko'rinishda yozamiz. Bunda $M(x,y) = a(x)y + b(x)$, $N(x,y) = -1$. Ravshanki,

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = a(x), \qquad \frac{a(x)}{N} = -a(x).$$

Demak, $\mu = \mu(x).\mu(x) = Ce^{\int_{x_0}^{x} \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx}$ *shu tenglamaga ko'ra:* $\mu(x) = e^{-\int_{x_0}^{x} a(\delta)d\delta}$ shunday qilib ,birinchi tartibli chiziqli differensial tenglamaning integrallovchi ko'paytuvchi ko'rinishida bo'ladi.

Ushbu $(xy^2 - y)dx + xdy = 0$ differinsial tenglama to'liq differinsialli emas , chunki:

$\frac{\partial M}{\partial y} = 2xy - 1$, $\frac{\partial N}{\partial x} = 1$, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{2(xy-1)}{xy^2 - y} = \frac{2}{y}$. Demak, $\mu = \mu(y)$ .Shuning uchun

$\mu(y) = e^{-\int_{y_0}^{y} \frac{2}{y} dy} = e^{-2ln\frac{y}{y_0}} = \left(\frac{y}{y_0}\right)^{-2}$ yoki $y_0 = 1$ *deb* $\mu(y) = \frac{1}{y^2}$ integrallovchi ko'paytuvchiga ega bo'lamiz.

Berilgan tenglamani integrallash jarayonini oxiriga yetkazib qo'yamiz .Uni $\frac{1}{y^2}$ ga ko'paytirib, to'liq differinsialli tenglamani hosil qilamiz:

$$\left(x-\frac{1}{y}\right)dx+\frac{x}{y^2}dy=0$$

Bu tenglama uchun

$\frac{x^2}{2}-\frac{x}{y}$=C

Umumiy yechim bo'ladi.

v) $\mu(x,y)=\mu_1(x)\mu_2(y)$ deylik . $M(x,y)dx+N(x,y)dy=0$ differinsial tenglama shu ko'rinishda integrallovchi ko'paytuvchiga ega bo'lish shartini chiqaramiz .

$\mu_1(x,y)=\mu(x,y)\Phi(U),\Phi(U(x,y))\in C^1(G)$ dan

$$M\cdot\frac{1}{\mu_2}\frac{d\mu_2}{dy}-N\frac{1}{\mu_1}\frac{d\mu_1}{dx}=\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}$$

Yoki

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=N\varphi_1(x)-M\varphi_2(y)$$

ega bo'lamiz , bu yerda

$\varphi_2(y)=\frac{1}{\mu_2}\frac{d\mu_2}{dy},\ \varphi_1(y)=\frac{1}{\mu_1}\frac{d\mu_1}{dx}$

Shunday qilib , agar $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}$ ifoda

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=N\varphi_1(x)-M\varphi_2(y)$$

Ko'rinishida yozilishi mumkin bo'lsa ,u xolda $M(x,y)dx+N(x,y)dy=0$ tenglama $\mu=\mu_1(x)\mu_2(y)$ ko'rinishida integrallovchi ko'paytuvchiga ega bo'ladi ,bunda $\mu_1(x)\ va\ \mu_2(y)\ funksiyalar\ \varphi_2(y)=\frac{1}{\mu_2}\frac{d\mu_2}{dy},\ \varphi_2(y)=\frac{1}{\mu_1}\frac{d\mu_1}{dx}$ formulalar bilan topiladi:

$$\mu_1(x)=e^{\int\varphi_1(x)dx},\mu_2(x)=e^{\int\varphi_2(x)dx},$$

Ushbu $(y^4-4xy)dx+(2xy^3-3x^2)dy=0,\ x>0,y>0,\ \frac{1}{4}y^3<x<\frac{2}{3}y^3$ differinsial tenglama integrallansin.

Bu to'liq differinsialli emas ,chunki $M=y^4-4xy,\ N=2xy^3-3x^2\ va\ \frac{dM}{dy}=4y^3-4x,$ $\frac{dN}{dx}=2y^3-6x$ munosabatlardan $\frac{dM}{dy}\neq\frac{dN}{dx}$ mana shu tengsizlik kelib chiqadi.

Berilgan differinsial tenglama $\mu(x,y)=\mu_1(x)\mu_2(y)$ ko'rinishdagi integrallovchi ko'paytuvchiga ega , chunki $\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=4y^3-4x-(2y^3-6x)=(2xy^3-3x^2)\cdot\frac{2}{x}-(y^4-4xy)\frac{2}{x}=N\frac{2}{x}-M\frac{2}{x}.$

Bundan $\varphi_1(x)=\frac{2}{x}$ $\varphi_2(y)=\frac{2}{y}$, va

$$\mu_1(x)=e^{\int\frac{2}{x}dx}=x^2,\mu_2(y)=e^{\int\frac{2}{y}dy}=y^2,$$

g) $M(x,y)dx+N(x,y)dy=0$ differinsial tenglamada $M(x,y)$ va $N(x,y)$ funksiyalar G sohada aniqlangan, differinsiallanuvchi va m-tartibli bir jinsli bo'lsin . U holda tenglama

$$\mu(x,y)=\frac{1}{xM+yN}$$

ko'rinishida integrallovchi ko'paytuvchiga ega .Haqiqatdan $M(x,y)=x^mM\left(1,\frac{y}{x}\right),$

$$N(x,y)=x^mN\left(1,\frac{y}{x}\right),$$

va $\quad x^m M\left(1,\frac{y}{x}\right)dx + x^m N\left(1,\frac{y}{x}\right)dy = 0$

Agar $\frac{y}{x} = u$ desak $x^m M(1,u)dx + x^m N(1,u)(xdu + udx) = 0$ yoki

$$[x^m M(1,u)dx + ux^m N(1,u)]dx + x^{m+1}N(1,u)du = 0$$

Bunday integrallovchi ko'paytuvchi uchun

$$\mu_1(x,y) = \frac{1}{x^m[M(1,u) + uN(1,u)]}$$

formula kelib chiqadi .Berilgan tenglama uchun avvalgi belgilashlarga qaytib $\mu(x,y) = \frac{1}{xM+yN}$ formulani hosil qilamiz.

c) $M(x,y)dx + N(x,y)dy = 0$ differinsial tenglamaning ixtiyoriy integrallovchi ko'paytuvchisi ushbu $\mu_1(x,y) = \Phi(U)\mu(x,y)$ formula bilan beriladi ,bunda $\mu(x,y)$ biror integrallovchi ko'paytuvchi , Φ esa $M(x,y)dx + N(x,y)dy = 0$ tenglama integrali U ning ixtiyoriy uzluksiz funksiyasi .Differinsial tenglamaning ixtiyoriy integrallovchi ko'paytuvchisi $\mu_1(x,y) = \Phi(U)\mu(x,y)$ formula bilan yozish mumkin. Bu formula integrallovchi ko'paytuvchini topish yuqoridagi usullardan farq qiladigan usulini qo'llashga olib keladi.Yangi usul quyidagidan iborat $M(x,y)dx + N(x,y)dy = 0$ tenglamani shartli ravishda ikkiga bo'lamiz:$[M_1(x,y)dx + N_1(x,y)dy] + [M_2(x,y)dx + N_2(x,y)dy]$=0 bunda $M_1 + M_2 = M$ $N_1 + N_2 = N$. So'ngra ushbu $M_1dx + N_1dy = 0$, $M_2dx + N_2dy = 0$ tenglamalarni ayrim-ayrim ko'ramiz.Albatta, bu differensial tenglamalar uchun integrallovchi ko'paytuvchini nisbatan osonlik bilan topa olamiz , deb hisoblaymiz . Tegishli tenglamalarning integrallovchi ko'paytuvchilarni mos ravishda μ_2 va μ_2 integrallarini esa $U_1\ va\ U_2$ deylik.

U holda yuqoridagi formulaga asosan har bir differinsial tenglama ixtiyoriy integrallovchi ko'paytuvchini $\mu_1^* = \mu_1\Phi_1(U_1)$, $\mu_2^* = \mu_2\Phi_2(U_2)$, ko'rinishida yozish mumkin . Φ_1 va Φ_2 larning ixtiyoriyligidan foydalanib, ularni shunday tanlaymizki , ushbu $\mu_1^* = \mu_2^* = \mu$

Munosabatlar o'rinli bo'lsin .U holda μ funksiya berilgan $M(x,y)dx + N(x,y)dy = 0$ differinsial tenglama uchun integrallovchi ko'paytuvchi bo'ladi . Amalda $\Phi_1\ yoki\ \Phi_2$ funksiyani 1 ga teng qilib olish mumkin .

Misol. Ushbu $(xy^2 + y^4)dx + (x^2 - xy^3)dy = 0$, $x > 0,\ y > 0,\ x > y^3$ *differensial* tenglamaning integrallovchi ko'paytuvchisi topilsin .

Bu tenglamani

$$d(xy) + \frac{y^3}{x}(ydx - xdy) = 0$$

Ko'rinishda yozish mumkin . Undan μ_1^*=$\mu_1\Phi_1(xy) = \Phi_1(xy).\frac{y^3}{x}(ydx - xdy) = 0$ tenglama uchun $\mu_2 = \frac{1}{x^2y^2}$ ekanini v bo'limdagi usul bilan isbotlash mumkin . $\mu_1^* = \mu_2^*$ bo'lishi uchun Φ_2=1 desak , $\mu_2^* = \Phi_1\ (xy) = \frac{1}{x^2y^2} = \mu$ kelib chiqadi . Demak , $\mu = \frac{1}{x^2y^2}$ funksiya berilgan differensial tenglama uchun integrallovchi ko'paytuvchi bo'ladi.

Xulosa qilib shuni aytish mumkinki talabalarni interal ko'paytuvchini topishda yuqoridagi usullardan foydalanishi to'liq differensila tenglama bo'lmagan tenglamalarni to'liq differensial tenglamalarga keltirishda kata qulayliklar hosil qiladi.

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