# SOLVING MULTIDIMENSIONAL PROBLEMS WITH A WEAK APPROXIMATION METHOD

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## ANNOTATION

This paper examines the problem of solving multidimensional problems using the weak approximation method.

**Keywords:** Cauchy problem, approximation, oscillation, displacement operator.

## INTRODUCTION

Until now, we have considered the fractional step method as a method of constructing economical differential schemes. we show the application of the method of fractional steps to differential equations. in this sense, it can be interpreted as a weak method of approximation with a special appearance. let's start by looking at simple examples.

1. For the following cauchy problem:

$$\frac{dx}{dt} = f(t) \equiv 1,\; x(0) = 0, \tag{2.1}$$

We use the following fractional step differential scheme:

$$\frac{x^{n+1/2} - x^n}{\tau} = 1, \tag{2.2a}$$

$$\frac{x^{n+1} - x^{n+1/2}}{\tau} = 0 \tag{2.2b}$$

$$x^0 = 0. \tag{2.2v}$$

the following whole step-by-step scheme corresponds to this::

$$\frac{x^{n+1} - x^n}{\tau} = 1,\; x^0 = 0. \tag{2.3}$$

It follows that (2.2) scheme (2.1) provides a clear solution to the problem. However, the scheme (2.2) can be interpreted as follows: in the first half step (2.2a) we solve the following equation:

$$\frac{1}{2}\frac{dx}{dt} = 1. \qquad (2.4a)$$

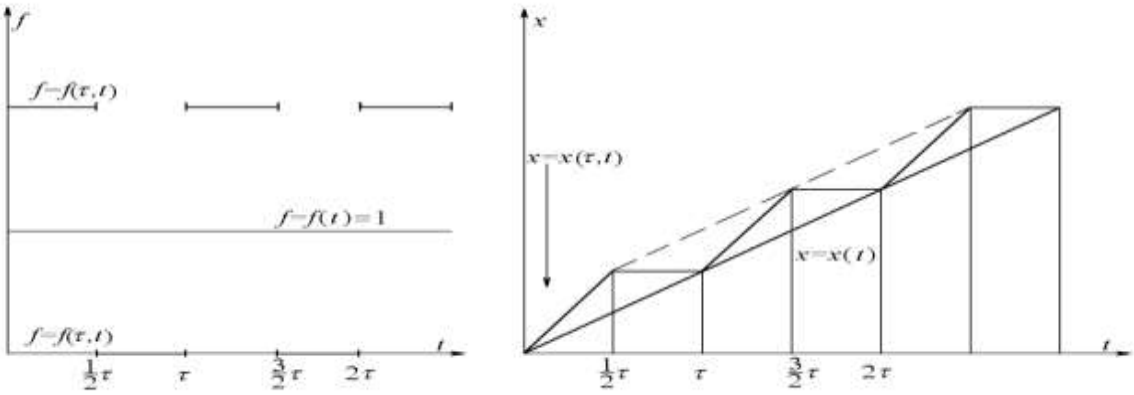



Fig 1., Comparison of $f(t) \equiv 1$, $f(\tau,t)$ functions and compatibility of $x(t)$, $x(\tau,t)$ curvilinear integrals.

In the second (2.2b) half-step, we solve the following equation:

$$\frac{1}{2}\frac{dx}{dt} = 0. \qquad (2.4b)$$

In general, the following equation is solved:

$$\frac{dx}{dt} = f(\tau,t),\ x^0 = 0, \qquad (2.5)$$

where function $f(\tau,t)$ is defined as follows:

$$f(\tau,t) = 2,\ n\tau < t \le (n+1/2)\tau,$$
$$f(\tau,t) = 0,\ (n+1/2)\tau < t \le (n+1)\tau.$$

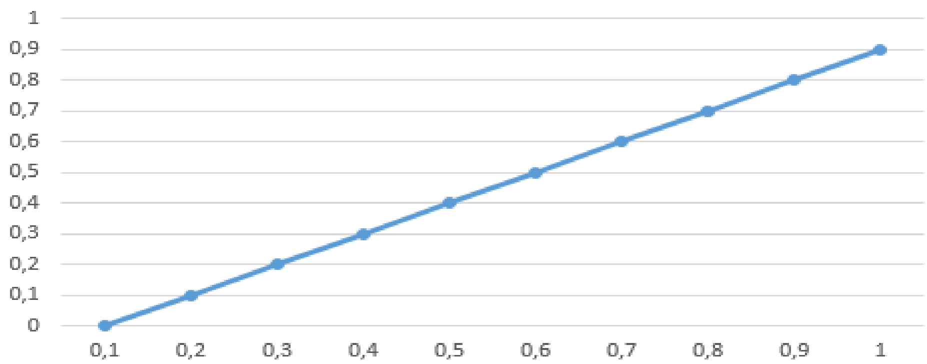
Figure 1 shows a comparison of the graph of , functions and the solution of in (2.1) and in (2.5).

It's not hard to see that functions are weakly compatible with functions

$$\int_{t_1}^{t_2}[f(\tau,s) - f(s)]ds \to 0,\ \tau \to 0,\ t_1,t_2 \text{ - optional}, \qquad (2.6)$$

(2.5) according to solution (2.1) when the solution is strongly compatible.

Problem 1 of the weak approximation method



1. For the following Cauchy issue:

$$\frac{dx}{dt} + ax = 0,\ x(0) = 1,\ a > 0 \tag{2.7}$$

We use the following differential scheme:

$$\frac{x^{n+1/2} - x^{n}}{\tau} + ax^{n} = 0, \tag{2.8a}$$

$$\frac{x^{n+1} - x^{n+1/2}}{\tau} = 0, \tag{2.8b}$$

$$x^{0} = 1. \tag{2.8v}$$

Scheme (2.8) can also be used as a solution of the following equation:

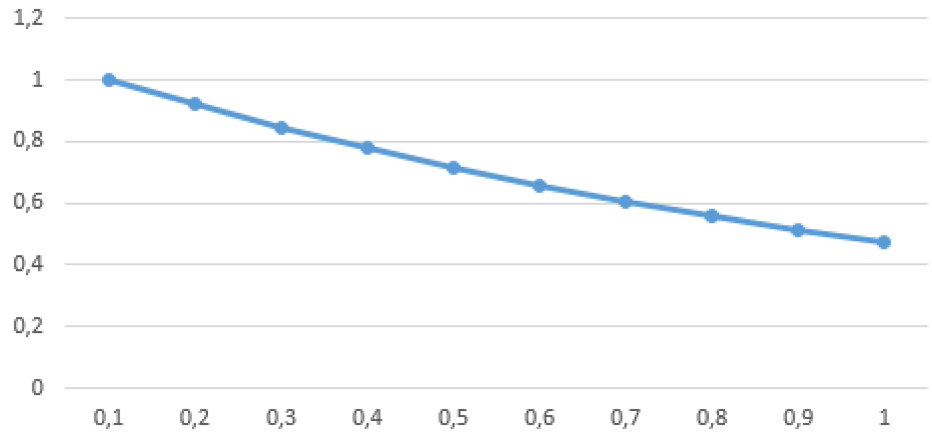
$$\frac{dx}{dt} + a(\tau, t)x = 0,\ x(0) = 1, \tag{2.9}$$

here

$$a(\tau,t) = 2a;\ n\tau < t \le (n+1/2)\tau;$$
$$a(\tau,t) = 0;\ (n+1/2)\tau < t \le (n+1)\tau, \tag{2.10}$$

$a(\tau,t)$ (2.6) approximates $a(t) \equiv a$ weakly in the form of equation, (2.9) strongly corresponds to solution (2.7).

Problem 2 of the weak approximation method




2. For the displacement equation, we consider the following Cauchy problem:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x_1} + \frac{\partial u}{\partial x_2} = 0;\ u(x_1, x_2, 0) = u_0(x_1, x_2). \tag{2.11}$$

The solution is as follows:

$$u(x_1, x_2, t) = u_0(x_1 - t, x_2 - t) = T_{-1}(t)T_{-2}(t)u_0(x_1, x_2), \tag{2.12}$$

where, $T_{-1}(t)$, $T_{-2}(t)$ shift operators have the following meaning:

$$T_{-1}(t)f(x_1, x_2) = f(x_1 - t, x_2);$$
$$T_{-2}(t)f(x_1, x_2) = f(x_1, x_2 - t). \tag{2.13}$$

Thus, the operator of solution $S(t)$ of equation (2.11) has the following form:

$$S(t)=T_{-2}(t)T_{-1}(t). \tag{2.14}$$

Retaining the initial data function $u(x_1,x_2,0)=u_0(x_1,x_2)$, we replace Equation (2.11) with Equation with Vibration Coefficient:

$$\frac{\partial u}{\partial t}+f_1(\tau,t)\frac{\partial u}{\partial x_1}+f_2(\tau,t)\frac{\partial u}{\partial x_2}=0, \tag{2.15}$$

where

$$\begin{aligned} &f_1(\tau,t)=2;\ f_2(\tau,t)=0; n\tau<t\le(n+1/2)\tau;\\ &f_1(\tau,t)=0;\ f_2(\tau,t)=2;(n+1/2)\tau<t\le(n+1)\tau. \end{aligned} \tag{2.16}$$

The solution of equation (2.15) in the range from $n\tau$ to $(n+1)\tau$ is as strong as the solution of the following series of equations:

$$\frac{\partial u}{\partial t}+2\frac{\partial u}{\partial x_1}=0;\ n\tau<t\le(n+1/2)\tau, \tag{2.17a}$$

accordingly

$$\frac{\partial u}{\partial t}+2\frac{\partial u}{\partial x_2}=0;\ (n+1/2)\tau<t\le(n+1)\tau. \tag{2.17b}$$

The displacement operator of equation (2.17a) is:

$$S_1(t+\frac{\tau}{2},t)=S_1(\frac{\tau}{2})=T_{-1}(\tau), \tag{2.18a}$$

The displacement operator of equation (2.17b) is:

$$S_2(t+\frac{\tau}{2},t)=S_2(\frac{\tau}{2})=T_{-2}(\tau). \tag{2.18b}$$

Then we have the following expression for $S_\tau(t+\tau,t)=S_\tau(\tau)$ shift operator in equation (2.15):

$$S_\tau(\tau)=S_2(\frac{\tau}{2})S_1(\frac{\tau}{2})=T_{-2}(\tau)T_{-1}(\tau)=S(\tau),$$

where $S(\tau)$ is the displacement operator of equation (2.11).

(2.15) weakly approximates equation (2.11), while operator $S_\tau(t,0)$ in equation (2.15) approximates strongly to solution $S(t,0)$ in equation (2.11) by decreasing for the last $t=n\tau$ .

3- For the thermal conductivity equation

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_2^2} \tag{2.19}$$

together with the oscillation coefficients we construct the following equation:

$$\frac{\partial u}{\partial t}=a_1(\tau,t)\frac{\partial^2 u}{\partial x_1^2}+a_2(\tau,t)\frac{\partial^2 u}{\partial x_2^2}, \tag{2.20}$$

where

$$\begin{aligned} &a_1(\tau,t)=2, a_2(\tau,t)=0; n\tau<t\le(n+1/2)\tau,\\ &a_1(\tau,t)=0, a_2(\tau,t)=2; (n+1/2)\tau<t\le(n+1)\tau. \end{aligned} \tag{2.21}$$

Then again, the following relationship can be easily established:

$$S(\tau)=S_2(\frac{\tau}{2})S_1(\frac{\tau}{2})=S_\tau(\tau), \tag{2.22}$$

where $S(\tau)$ is the displacement operator of equation (2.19); $S_i(\tau)$ is the displacement operator of the following equation:

$$\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x_i^2},\ i=1,2; \tag{2.23}$$

$S_\tau(\tau)$ - The displacement operator of equation (2.20).

The two-dimensional equation of thermal conductivity let

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_2^2}+f(x,t),$$

$$\overline{D}=\{0\le x_1\le 1, 0\le x_2\le 1, 0\le t\le T\},$$

$$u(x,0)=u_0(x),$$

$$u\big|_\Gamma=\mu(x,t)$$

be given. There are a total of 5 conditions, including 1 initial and 4 boundary conditions. Assuming that the main function is $u(x_1,x_2,t)=e^{A(x_1+x_2+t)}$, the conditions are as follows:

$$\begin{cases} u(x_1,x_2,0)=e^{A(x_1+x_2)},\\ u(0,x_2,t)=\mu_1(x_2,t)=e^{A(x_2+t)},\\ u(1,x_2,t)=\mu_2(x_2,t)=e^{A(1+x_2+t)},\\ u(x_1,0,t)=\mu_3(x_1,t)=e^{A(x_1+t)},\\ u(x_1,1,t)=\mu_4(x_1,t)=e^{A(x_1+1+t)}. \end{cases}$$

$$\Lambda_1 y=\frac{y(x_1-h_1,x_2)-2y(x_1,x_2)+y(x_1+h_1,x_2)}{h_1^2},$$

$$\Lambda_2 y=\frac{y(x_1,x_2-h_2)-2y(x_1,x_2)+y(x_1,x_2+h_2)}{h_2^2}.$$

$$\begin{cases} \dfrac{y^{n+1/2}-y^{n}}{0.5\tau}=\Lambda_1 y^{n+1/2}+\Lambda_2 y^{n}+\varphi^{n}, \\ \dfrac{y^{n+1}-y^{n+1/2}}{0.5\tau}=\Lambda_1 y^{n+1/2}+\Lambda_2 y^{n+1}+\varphi^{n}, \\ y(x,0)=u_0(x), \\ i_2=0, i_2=N_2, y^{n+1}=\mu^{n+1}, \\ i_1=0, i_1=N_1, y^{n+1/2}=\overline{\mu}. \end{cases}$$

where $\overline{\mu}=\frac{1}{2}(\mu^{n+1}+\mu^{n})-\frac{\tau}{4}\Lambda_2(\mu^{n+1}-\mu^{n})$.

$$f(x,t)=\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x_1^2}-\frac{\partial^2 u}{\partial x_2^2}=Ae^{A(x_1+x_2+t)}[1-2A].$$

All of this is illustrated below and solved by the method:

$$\frac{1}{h_1^2}\overline{y}_{i_1-1}-2\left(\frac{1}{h_1^2}+\frac{1}{\tau}\right)\overline{y}_{i_1}+\frac{1}{h_1^2}\overline{y}_{i_1+1}=-F_{i_1}, \tag{3.4}$$

$$i_1=1,2,\ldots,N_1-1, i_1=0, N_1\ da\ \overline{y}=\overline{\mu},$$

$$\frac{1}{h_2^2}\hat{y}_{i_2-1}-2\left(\frac{1}{h_2^2}+\frac{1}{\tau}\right)\hat{y}_{i_2}+\frac{1}{h_2^2}\hat{y}_{i_2+1}=-\overline{F}_{i_2}, \tag{3.5}$$

$$i_2=1,2,\ldots,N_2-1, i_2=0, N_2\ da\ \overline{y}=\overline{\mu}.$$

We have now obtained the following results through a computational experiment:

N1=5; N2=5

Initial condition values:

| | | | | | |
|---|---|---|---|---|---|
| 1.0 | 1.010050167084168 | 1.0202013400267558 | 1.030454533953517 | 1.0408107741923882 | 1.0512710963760241 |
| 1.010050167084168 | 1.0202013400267558 | 1.030454533953517 | 1.0408107741923882 | 1.0512710963760241 | 1.0618365465453596 |
| 1.0202013400267558 | 1.030454533953517 | 1.0408107741923882 | 1.0512710963760241 | 1.0618365465453596 | 1.0725081812542165 |
| 1.030454533953517 | 1.0408107741923882 | 1.0512710963760241 | 1.0618365465453596 | 1.0725081812542165 | 1.0832870676749586 |
| 1.0408107741923882 | 1.0512710963760241 | 1.0618365465453596 | 1.0725081812542165 | 1.0832870676749586 | 1.0941742837052104 |
| 1.0512710963760241 | 1.0618365465453596 | 1.0725081812542165 | 1.0832870676749586 | 1.0941742837052104 | 1.1051709180756477 |

Solutions of the boundary condition 1:

| | | | | |
|---|---|---|---|---|
| 1.0105553184450264 | 1.0207115682431935 | 1.030969890048781 | 1.0413313097025174 | 1.0517968633550032 |
| 1.0110607224447195 | 1.0212220516375285 | 1.0314855038865227 | 1.0418521055454795 | 1.0523228932832038 |
| 1.0115663792095986 | 1.021732790337382 | 1.0320013755956459 | 1.0423731618514736 | 1.0528491862921334 |

Solutions of the boundary condition 2:

| | | | | |
|---|---|---|---|---|
| 1.062367597570325 | 1.073044569430713 | 1.0838288466422508 | 1.0947215076416466 | 1.1057236417040774 |
| 1.0628989141871952 | 1.0735812258683575 | 1.0843708965667604 | 1.0952690052584655 | 1.1062766417634236 |
| 1.0634304965287997 | 1.0741181507013138 | 1.0849132175839997 | 1.0958167766925413 | 1.1068299183919357 |

Solutions of the boundary condition 3:

| | | | | |
|---|---|---|---|---|
| 1.0105553184450264 | 1.0207115682431935 | 1.030969890048781 | 1.0413313097025174 | 1.0517968633550032 |
| 1.0110607224447195 | 1.0212220516375285 | 1.0314855038865227 | 1.0418521055454795 | 1.0523228932832038 |
| 1.0115663792095986 | 1.021732790337382 | 1.0320013755956459 | 1.0423731618514736 | 1.0528491862921334 |

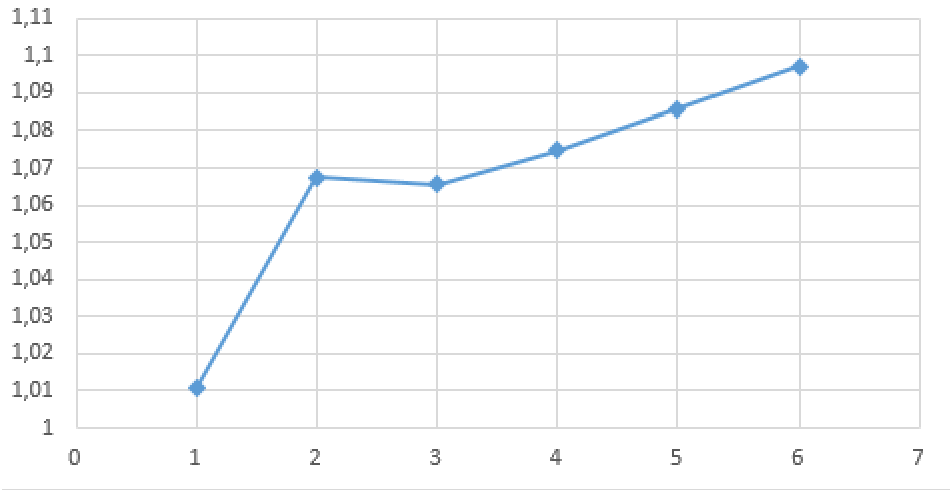
Solutions of the boundary condition 4:

| | | | | |
|---|---|---|---|---|
| 1.062367597570325 | 1.073044569430713 | 1.0838288466422508 | 1.0947215076416466 | 1.1057236417040774 |
| 1.0628989141871952 | 1.0735812258683575 | 1.0843708965667604 | 1.0952690052584655 | 1.1062766417634236 |
| 1.0634304965287997 | 1.0741181507013138 | 1.0849132175839997 | 1.0958167766925413 | 1.1068299183919357 |

Approximate solution:

y_2[ 0 ]= 1.0107156140112834
y_2[ 1 ]= 1.0674482059583017
y_2[ 2 ]= 1.0655469152255599
y_2[ 3 ]= 1.0747297719546063
y_2[ 4 ]= 1.0857926874517507
y_2[ 5 ]= 1.0971389239568121

Approximate solution graph

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