

IDEAS FOR THE CONSTRUCTION OF CHARACTERISTIC-GRID METHODS AND THE ANALYSIS OF DIFFERENTIAL SCHEMES IN THE SPACE OF INDETERMINATE COEFFICIENTS

Raufov Azizbek Anvar o'g'li

Jonqobilov Mirjalol

ANNOTATION

This article discusses the ideas of building characteristic-grid methods and the analysis of differential schemes in the space of indeterminate coefficients.

Keywords: differential scheme, indeterminate coefficients, linear equation, characteristic-grid method.

INTRODUCTION

The basic ideas of the construction of the characteristic-grid method are shown in the example of the solution of the simple linear equation of displacement in accordance with the initial and boundary conditions:

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = f(x, t), \quad c = \text{const} > 0. \quad (1)$$

It is known that (1) has a characteristic defined by the following characteristic equation:

$$\frac{dx}{dt} = a$$

and it becomes a simple differential equation (ODT):

$$\frac{du}{d\xi} = f(\xi).$$

Grid methods are based on the addition of a grid template to the integration area of a differential grid equation, that is, the set of grid nodes used to replace the differential analog (approximation) differential equation (in a small area). Approximation analysis usually uses the Taylor series spread of a precise solution of a differential problem in a grid. Let's look at both the grid and the characteristic class method.

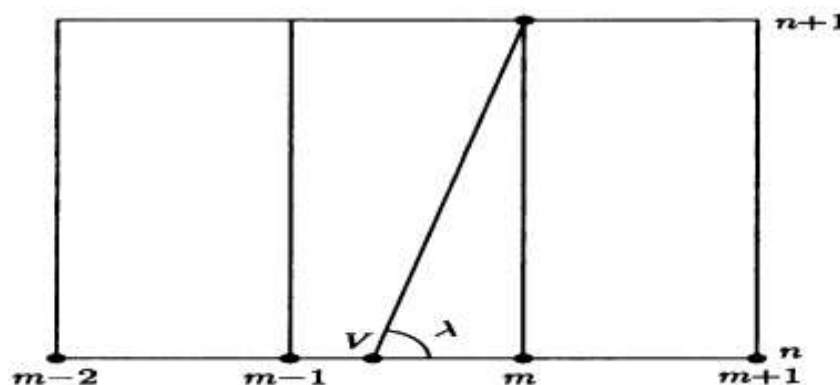


Fig. 1.

In the field of integration we include the following difference grid:

$$t^n = n \cdot \tau, x_m = m \cdot h; n = 0, 1, 2, 3, \dots; m = 0, \pm 1, \pm 2, \pm 3, \dots$$

(1) We denote the values of the solution by u_m^{n+1} at the nodes of the network. To proceed to the next temporary step of the grid, we use an open five-dot template (Figure 1). Value u_m^n must be calculated over time using the values in layers u and n . We transfer the characteristic from $n+1, m$ points to the intersection of λ slopes to V points with t^n layers. Then the value characteristic u_m^{n+1} is represented by u_v^n , for example

$$u_m^{n+1} = u_v^n + \frac{\tau}{2} (f_m^{n+1} + f_v^n) + O(\tau^2). \tag{2}$$

We obtain expression (2) as an approximation of a simple differential equation by the characteristics of the trapezoidal method. To find the value of u_v^n on the points of layer t^n , we have different differential schemes to determine u_m^{n+1} in the given template by performing this or that integration.

Space of indefinite coefficients.

We write the difference scheme as follows:

$$u_m^{n+1} = \sum_{k=-2}^1 \alpha_k \cdot u_{m+k}^n, \tag{3}$$

here we look for the coefficients of the approximation of the α_k coefficients by spreading the u_m^{n+1}, u_{m+k}^n to the Taylor series around (t^n, x_m) point. By leaving two coefficients blank (for example, α_{-2}, α_0), we obtain a $O(\tau + h)$ -order approximation condition:

$$\alpha_{-1} = \frac{1}{2} (1 + \sigma - 3 \cdot \alpha_{-2} - \alpha_0),$$

$$\alpha_1 = \frac{1}{2} (1 - \sigma + \alpha_{-2} - \alpha_0).$$

We take empty coefficients as starting points for coordinates in linear space with Euclidean metrics. Each point in this space corresponds to a differential scheme with first-order approximation. You can also define multiple $O(\tau + h^2)$ -order schemes

$$\alpha_0 = 1 - \sigma + 3 \cdot \alpha_{-2} \tag{4}$$

and in this template a single third-order approximation scheme can be defined in $O(\tau + h^3)$ order:

$$\alpha_{-2} = \frac{\sigma \cdot (\sigma - 1)}{6}. \tag{5}$$

The remaining coefficients are found using (4) and (5).

(α_{-2}, α_0) we enter the space of the coefficients. Then there is an arbitrary point in this space $O(\tau + h)$ is an orderly approximation differential scheme. (4) separates many schemes of order $O(\tau + h^2)$ in direct space (Fig. 2) because it contains a single point of approximation $O(\tau + h^3)$. There must also be a $O(\tau^2 + h^2)$ -order approximation point.

We record a number of chimes, for example, we apply the indeterminate coefficients of the spectral sign (Neumann background) to the differential scheme $\sigma = 0.5$. (3). In the space of indefinite coefficients, a curve is formed that defines the stability limit of the differential circuits.

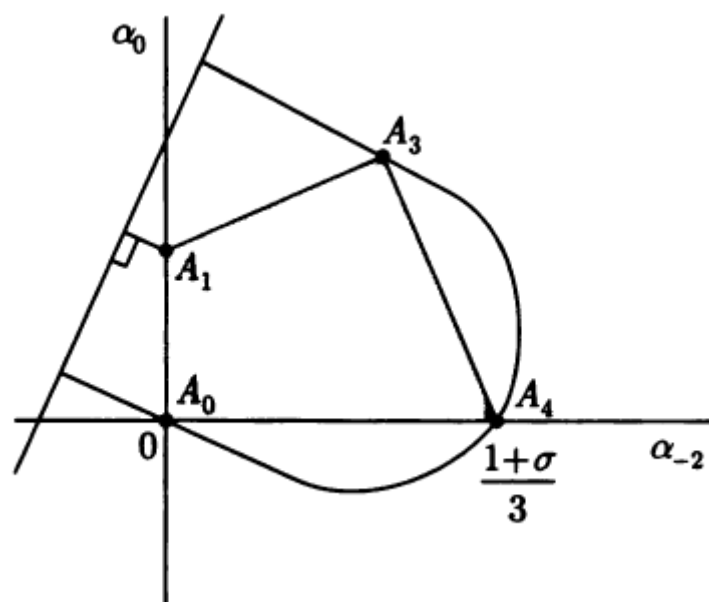


Fig 2.

For first-order schemes, we write the first differential approximation:

$$u'_t + au'_x = \frac{h^2}{\tau} (1 - \sigma^2 - \alpha_0 + 3\alpha_{-1})u''_{xx}.$$

You can specify many schemes, such as $\alpha_\mu \geq 0$ this is a dashed polygon in monotone schemes (Figure 2). Monotonous schemes include schemes with less approximation error. Given a polygonal σ point, it lies in the nearest second-order approximation diagrams.

Let's end the example with the linear equation of the displacement model:

$$u'_t + cu'_x = 0.$$

The selected template displays an arbitrary differential scheme as follows:

$$u_m^{n+1} = \sum_{\mu \in III} \alpha_\mu u_{m+\mu}^n.$$

Many positive approximation schemes do not intersect with many first-order approximation schemes according to S. K. Godunov's theorem.

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