

WAYS OF SEPARATION OF ROOTS AND METHODS OF DIVIDING INTERMEDIATE

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ANNOATION

Finding the roots of a zero-dimensional polynomial system is a fundamental problem with a wide range of applications, including algebraic geometry, computer graphics, and computer-aided geometric design. The development of robust and validated algorithms in particular and efficient methods for determining isolating zones for all roots of polynomial systems are required. This article discusses about ways of separation of roots and methods of dividing intermediate.

Keywords: separation, roots, methods, approaches, diving intermediate, demensions, tools.

INTRODUCTION

Non-simple roots should likewise be handled by such methods which mixed with efficient symbolic computations, so that the total result can be verified? In this context, we regard our key contribution as developing a revolutionary certification mechanism. The first thread is inspired by elimination methods such as Multivariate resultants bases. Both are well-studied tools to obtain the solution set of a system with respect to a projection direction. However, both methods lead to polynomials with very large bitsizes (for intermediate results as well as for the final result), which causes a severe drawback regarding the performance. Therefore, our method computes the multivariate resultant (with some hidden variable) only in several prime fields Z_p and completely avoids Chinese Remaindering. In particular, all symbolic computations are performed using single precision arithmetic. This method yields a lower bound on the number of projected solutions. Although this bound is very likely to match the exact number in practice, this cannot be certified without further knowledge. The second thread follows an Exclusion and Subdivision method. It keeps on subdividing regions that may contain solutions (from now on, we call such connected regions clusters), whereas regions that doubtlessly do not contain a solution are discarded. As a simple exclusion method we use interval arithmetic. The primary idea is to run two computing threads in parallel. This technique uses Partial Fraction Expansion to split up a complicated fraction into forms that are in the Laplace Transform table. As you read through this section, you may find it helpful to refer to the review section on partial fraction expansion techniques. The text below assumes you are familiar with that material.

SEPARATION OF ROOTS. HOW TO DIVIDE INTERMEDIATE

The process of approximate solution of equations is divided into two stages:

- 1) Separation of roots;
- 2) Find the roots with given accuracy.

$[a, b]$ at the intersection of the equation $f(x) = 0$

If there is no root other than, root is allocated. To separate the roots, cut $[a, b]$ into such incisions. Let these intersections have only one root of the equation. The roots can be distinguished by **graphic** and **analytic** methods.

Graphical separation of roots.

Method 1 This method is very simple and follows is done. In the Cartesian coordinate system, we draw a graph of the function $u = f(x)$ (this is us known from the high school curriculum). The points of intersection of this graph with the Ox axis

The roots you are looking for are (approximate). For example. x

Method 2 $f(x) = 0$ equation $f(x) = f$

$2(x)$ in the form. In the Cartesian coordinate system $f_1(x)$ and f We draw graphs of functions (x) .

If these curves intersect, a straight line from the point of intersection to the axis Ox (perpendicular). The resulting points (two points) are approximate solutions will be X in Figure 2

1 and x are approximate solutions of equation

Graphs to get more accurate solutions when solving equations with these methods you need to draw as accurately as possible and get a large scale. Still Roots cannot be calculated with high accuracy by graphical methods. By the graphical method. We determine the roots of the equation in a bounded section, ie the graph we can't get as big as we want and how many roots the equation has we can not answer. If the roots need to be found with high precision, another approximation methods should be used.

Analytical separation of roots. Analyze the roots of the equation $f(x) = 0$

Here are some theorems from the higher mathematics course without proof.

Theorem 1. If the function $f(x)$ is continuous in the intersection $[a, b]$, then the intersection

If we accept different sign values at the endpoints, then $[a, b]$ is the intersection

The equation $f(x) = 0$ has at least one root.

Theorem 2. If the function $f(x)$ is continuous and monotonous at the intersection $[a, b]$, assumes different sign values at the edge points of the intersection, then $[a, b]$ only one root of the equation $f(x) = 0$ lies in the cross section.

Theorem 3. If the function $f(x)$ is continuous in the intersection $[a, b]$ and the intersection assuming different sign values at the endpoints, $[a, b]$ within the intersection $f'(x)$

If the sign of the product does not change, then in the section $[a, b]$ only $f(x) = 0$ one root lies.

Note. 1) The function $u = f(x)$ is called monotone in a given interval, if this desired x belonging to the interval

HOW TO DIVIDE INTERMEDIATE

Assume that $f(x) = 0$ is one of the equations the root $[a, b]$ is separated in the cross section

Let Let us denote the length of the section $d = b - a$. Of the equation solution $= 0.001$ be found in precision.

that the root $[a, b]$ is inside $\{a < x < b\}$ with a n less than the approximate root obtained, we can take b as the approximate root obtained with the excess. If $d < 0.001$, the problem is solved and a and b are equal to $f(x) = 0$ given $= 0.001$

Since polynomial root solving is such an important problem in several fields, plenty of distinct approaches exist and many textbooks are dedicated to this subject. See, for instance, for introductions to symbolic approaches such as (sparse) resultants and methods based on eigenvalue computations or on rational univariate representation. Homotopy methods numerically track the continuous path of the known complex solutions of some trivial and appropriate polynomial system during a continuous deformation into the input system. Such methods, although very robust, lack the certification of their output in general. We recommend for a more comprehensive overview. Subdivision methods describe a further class of common tools. Algorithm of that kind profit from their efficiency and plainness. Most Implementations are using one of the numerous software packages for efficient interval arithmetic such as IntBis, ALIAS, IntLab or MPFI. Alternative variants, using the Bernstein basis and convexity properties of their coefficients, have been addressed. However, all these approaches lack to certify their results – in general, an approach stops when a certain subdivision depth is reached or each region contains a simple root, which can, for instance, be certified by the interval Newton test. So far, in case of multiple roots, all proposed methods have to go below a certain a-priori worst case root separation bound in order to certify that a region is isolating. Analytical separation of roots. In this case, the first-order product of the function is determined, the critical points of the function are determined by solving the equation. At critical points, the signal exchanges of the function are determined and find the critical points of the function. compile a table of values of the function and determine the intervals of signal exchange.

Graphical separation of roots. In this case, the equation is written visually using elementary functions. a table of values of functions is created and their graphs are drawn. the smallest interval in which the intersection points of the graphs of the functions lie is the interval at which the root of the equation lies. We write the function in question visually and divide it into functions.

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